

ECE 479/579

Principles of Artificial Intelligence

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ASSIGNMENT # 2

First Order Logic

1. (10 pts.) Prove that implication is transitive in the propositional calculus, that is, that $((P \Rightarrow Q) \wedge (Q \Rightarrow R)) \Rightarrow (P \Rightarrow R)$.

2. (10 pts.) The connective \otimes (exclusive or) is defined by the following truth table:

| X1 | X2 | X1 \otimes X2 |
|----|----|-----------------|
| F | F | F |
| F | T | T |
| T | F | T |
| T | T | F |

What wff containing only \neg , \wedge and \vee connectives is equivalent to $(X1 \otimes X2)$?

3. (20 pts.) Show that $(\exists ?z)(\forall ?x)[P(?x) \Rightarrow Q(?z)]$ and $(\exists ?z)[(\exists ?x)P(?x) \Rightarrow Q(?z)]$ are equivalent.

4. (10 pts.) Find the **mgu** of the set $\{P(?x, ?z, ?y), P(?w, ?u, ?w), P(A, ?u, ?u)\}$.

5. (20 pts.) The logical operator \Leftrightarrow is read "if and only if" $P \Leftrightarrow Q$ is defined to be equivalent to $(P \Rightarrow Q) \wedge (Q \Rightarrow P)$. Based on this definition, show that $P \Leftrightarrow Q$ is logically equivalent to $(P \vee Q) \Rightarrow (Q \wedge P)$:

- By using truth tables
- By a series of substitutions using the basic identities.

6. (10 pts.) Suppose that we represent "Sam is Bill's father" by FATHER(BILL, SAM) and "Harry is one of Bill's ancestors" by ANCESTOR(BILL, HARRY). Write a wff to represent "Every ancestor of Bill is either his father, his mother, or one of their ancestors."

7. (20 pts.) Attempt to unify the following pairs of expressions. Either show their most general unifiers or explain why they will not unify. In the following assume father(?x) is a function of ?x.

- $P(?x, ?y)$ and $P(A, ?z)$
- $P(?x, ?x)$ and $P(A, B)$
- $\text{Ancestor}(?x, ?y)$ and $\text{Ancestor}(\text{Bill}, \text{father}(\text{Bill}))$
- $\text{Ancestor}(?x, \text{father}(?x))$ and $\text{Ancestor}(\text{David}, \text{George})$
- $Q(?x)$ and $\neg Q(A)$
- $P(?x, A, ?y)$ and $P(?z, ?z, B)$