Synthesizing Discrete Event Controllers for Robotic Assembly by Automatic Construction of Qualitative Contact Models

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Contents

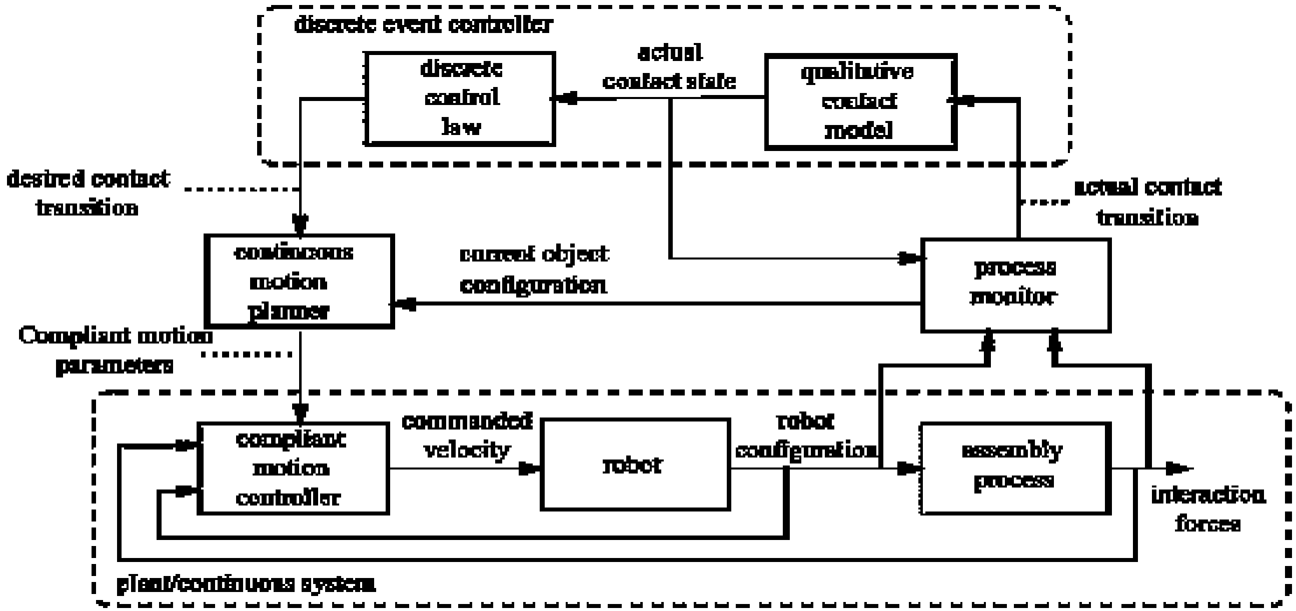
Automatic Assembly

- Position-based control
  - Easy to program
  - Expensive for precision position control
- Compliant motion control
  - Reduce the uncertainty
  - Requires less accurate and less expensive equipment
  - Hard to program

Qualitative Models

- Qualitative state adjacency graph
  - Expression: $G = (\Gamma, T)$
  - Γ : space of qualitative configuration representations
  - T : set of arcs connecting adjacent configuration states
- Augmented qualitative state adjacency graph
  - Expression: $G_a = (\Gamma, T, T_d)$
  - T_d : set of all desired transitions

Discrete Event Controller



- Discrete states: the qualitative distinct contact or configuration states
- Discrete events: perceived state transitions

Sets of Elementary Contacts – Contact Between Two Polyhedra

- A contact description is a set of elementary face-vertex, vertex-face and edge-edge contacts:

$$\gamma_{A,B} = \{f_i^A - v_j^B, \ldots, v_k^A - f_l^B, \ldots, e_m^A - e_n^B\}$$

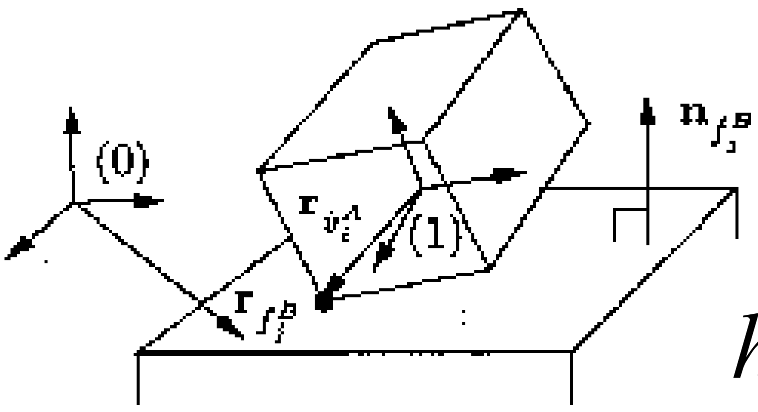
  The set of all syntactically correct contact description is denoted by $\Gamma_{A,B}$.
- A qualitative contact state is a geometrically feasible contact description $\gamma_{A,B}$. The set of all contact states is denoted by $\Gamma_{A,B}^c \subset \Gamma_{A,B}$.

Verifying of Contact Hypotheses for Polyhedra

- Problem statement: Given a hypothetical contact description, does there exist a configuration of the objects for which
  - the kinematic equality and bouding constraints imposed by the elementary contacts are satisfied.
  - the parts do not penetrate?

Criteria for Geometric Feasibility – Kinematic Constraints and Bounds

- Vertex-face contact

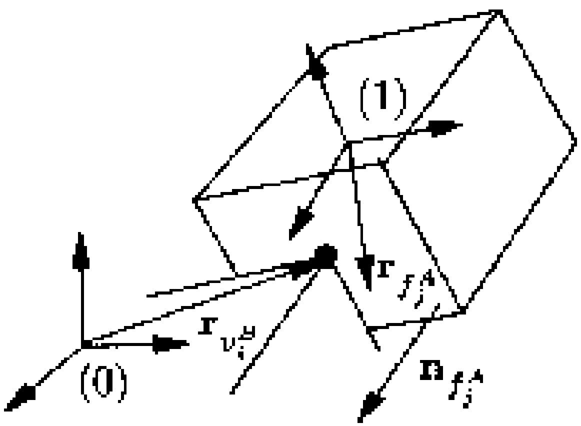


$$h^I_{eq}(f^B_j, v^A_i) \overset{def}{=} d^E_s(f^B_j, v^A_i) = 0$$

$$h^I_{bd}(f^B_j, v^A_i) \overset{def}{=} \prod_{f^B_c \in cv(f^B_j)} \left[\sum_{p^B_k \in bp(f^B_c)} \mu(-d^E_s(p^B_k, v^A_i)) \right] = 0$$

Criteria for Geometric Feasibility – Kinematic Constraints and Bounds (cont')

- Face-vertex contact

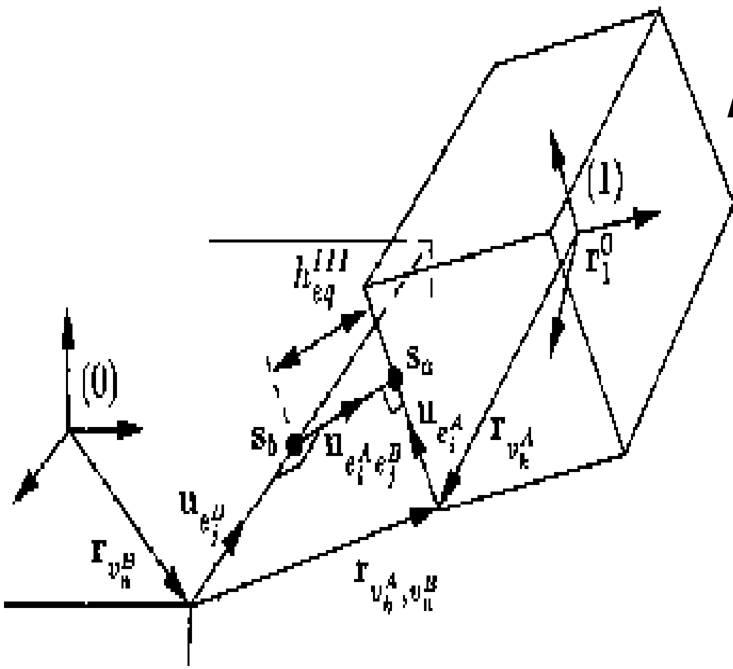


$$h_{eq}^{II}(v_i^B, f_j^A) \overset{def}{=} d_s^E(v_i^B, f_j^A) = 0$$

$$h_{bd}^{II}(v_i^B, f_j^A) \overset{def}{=} \prod_{f_c^A \in cv(f_j^A)} \left[\sum_{p_k^A \in bp(f_c^A)} \mu(-d_s^E(v_i^B, p_k^A)) \right] = 0$$

Criteria for Geometric Feasibility – Kinematic Constraints and Bounds (cont')

- Edge-edge contact



$$h_{eq}^{III}(d_j^B, e_i^A) \overset{def}{=} (n^0_{e_i^A e_j^B})^t (r^0_{v_k^A, v_n^B}) = 0$$

$$h_{bd}^{III}(e_j^B, e_i^A) \overset{def}{=} \mu(\alpha^B_{e_j^B e_i^A}) + \mu(l_{e_j^B} - \alpha^B_{e_j^B e_i^A})$$

$$+ \mu(\alpha^A_{e_j^B e_i^A}) + \mu(l_{e_i^A} - \alpha^A_{e_j^B e_i^A})$$

$$= 0$$

Criteria for Geometric Feasibility – Nonpenetration Condition

$$d_p^G(A,B) = \sum_{A_k \in cv(A)} [\sum_{B_l \in cv(B)} d_p^G(A_k, B_l)] = 0$$

$$d_p^G(A_k, B_l) = \begin{cases} (\rho_A + \rho_B)(1 - g(A_k, B_l)) & g(A_k, B_l) \leq 1 \\ 0 & g(A_k, B_l) > 1 \end{cases}$$

$$g(A_k, B_l) = \min \sigma$$

subject to $\quad A_k(\sigma) \underset{\sigma \in R^+}{\cap} B_l(\sigma) \neq 0$

Optimization Problem

$$\min \quad G = d_p^G(A, B)$$

subject to :

$$\forall (v_i^A - f_j^B) \in \gamma^{A,B} \qquad \begin{cases} h_{eq}^{I}(f_j^B, v_i^A) = 0 \\ h_{bd}^{I}(f_j^B, v_i^A) = 0 \end{cases}$$

$$\forall (f_j^A - v_i^B) \in \gamma^{A,B} \qquad \begin{cases} h_{eq}^{II}(v_i^B, f_j^A) = 0 \\ h_{bd}^{II}(v_i^B, f_j^A) = 0 \end{cases}$$

$$\forall (e_i^A - e_j^B) \in \gamma^{A,B} \qquad \begin{cases} h_{eq}^{III}(e_j^B, e_i^A) = 0 \\ h_{bd}^{III}(e_j^B, e_i^A) = 0 \end{cases}$$

Unconstrained Optimization Problem

$$
\begin{aligned}
G = (d_p^G)^2 &+ \sum_{(v_i^A - f_j^B)} [h_{eq}^I (f_j^B, v_i^A)]^2 + \sum_{(v_i^A - f_j^B)} h_{bd}^I (f_j^B, v_i^A) \\
&+ \sum_{(f_j^A - v_i^B)} [h_{eq}^{II} (v_i^B, f_j^A)]^2 + \sum_{(f_j^A - v_i^B)} h_{bd}^{II} (v_i^B, f_j^A) \\
&+ \sum_{(e_i^A - e_j^B)} [h_{eq}^{III} (e_j^B, e_i^A)]^2 + \sum_{(e_i^A - e_j^B)} h_{bd}^{III} (e_j^B, e_i^A)
\end{aligned}
$$

Solution of Optimization Problem

- The optimization problem can be solved numerically by simulating a viscoelastic dynamic system.
- The cost function G is a function of configuration of the mobile object, which is (R_1^0, r_1^0).
- A dynamic state equation: $\begin{bmatrix} v_1^0 \\ w_1^0 \end{bmatrix} = -B^{-1} \begin{bmatrix} f_G \\ \tau_G \end{bmatrix}$
- Gradient descent algorithms: $\begin{cases} \dot{R}_1^0 = \tilde{\omega}_1^0 R_1^0 \\ \dot{r}_1^0 = v_1^0 \end{cases}$

Experimental Results

Table 2: Experimental results for the dovetail assembly and the hollebol piece.

		$\gamma_{A,B}$	G	n_{ok}	n_{bad}	result
dovetail	1	$v_0^A - f_6^B, e_0^A - e_9^B$	0.0010	17	10	feasible
	2	$v_2^A - f_7^B, e_0^A - e_9^B$	0.0012	6	0	feasible
	3	$v_0^A - f_8^B, e_0^A - e_{10}^B$	3.8596	83	28	infeasible
hollebol piece	1	$v_5^A - f_5^B, e_{10}^A - e_{39}^B$	0.0002	8	1	feasible
	2	$v_9^A - f_{28}^B, e_{10}^A - e_{47}^B, e_{11}^A - e_{48}^B$	0.0013	49	9	feasible
	3	$f_{10}^A - v_8^B, f_{11}^A - v_{12}^B, f_{12}^A - v_{11}^B, f_{13}^A - v_{10}^B, f_{14}^A - v_9^B, v_5^A - f_5^B$	0.0006	13	4	feasible
	4	$v_5^A - f_{22}^B, v_6^A - f_{23}^B, v_7^A - f_{24}^B, v_8^A - f_{25}^B, v_9^A - f_{26}^B, f_0^A - v_8^B$	6.0511	51	9	infeasible

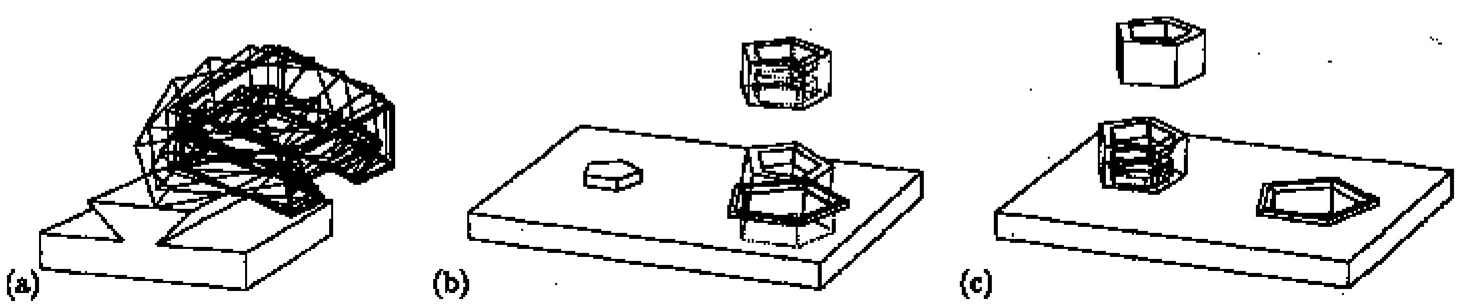


Figure 6: Verification of contact hypotheses for two different object pairs. Panel (a) shows verification of geometrically feasible contact hypothesis $\gamma_{A,B} = \{v_0^A - f_6^B, e_0^A - e_9^B\}$ (identical to Figure 1) for the dovetail assembly. Intermediate configurations are shown to emphasize that verification is a dynamic process. Panels (b) and (c) show testing of contact hypotheses $\gamma_{A,B} = \{v_9^A - f_{28}^B, e_{10}^A - e_{47}^B, e_{11}^A - e_{48}^B\}$ and $\gamma_{A,B} = \{f_{10}^A - v_8^B, f_{11}^A - v_{12}^B, f_{12}^A - v_{11}^B, f_{13}^A - v_{10}^B, f_{14}^A - v_9^B, v_5^A - f_5^B\}$ for the "hollebol" piece, respectively. For clarity, only the initial and final configuration are shown.

Discussion about Representing contact using Sets of Elementary Contacts

- Major problem: the lack of efficient methods to verify spatial adjacency of two states.
- Solution: search for a representation for which adjacency can be verified easily – a representation using feature interaction matrices.

Feature Interactions
- Vertex-Face Interaction

- The interactions between all vertices of A and all faces of B is given by a vertex-face interaction matrix:

$$
{}^{AB}\Phi^{vf} = [{}^{AB}\phi^{vf}_{ij}] = \begin{array}{c|cccc}
 & f_1^B & f_2^B & \cdots & f_{m_f^B}^B \\
\hline
v_1^A & {}^{AB}\phi^{vf}_{1,1} & {}^{AB}\phi^{vf}_{1,2} & \cdots & {}^{AB}\phi^{vf}_{1,m_f^B} \\
v_2^A & {}^{AB}\phi^{vf}_{2,1} & {}^{AB}\phi^{vf}_{2,2} & \cdots & {}^{AB}\phi^{vf}_{2,m_f^B} \\
\vdots & \vdots & \vdots & & \vdots \\
v_{m_v^A}^A & {}^{AB}\phi^{vf}_{m_v^A,1} & {}^{AB}\phi^{vf}_{m_v^A,2} & \cdots & {}^{AB}\phi^{vf}_{m_v^A,m_f^B}
\end{array}
$$

$$
{}^{AB}\phi^{vf}_{ij} = sign({}^{AB}h^{vf}_{ij}(R_1^0, r_1^0)) = sign(d_s^E(f_j^B, v_i^A))
$$

Feature Interactions (cont') - Face-Vertex Interaction

- The interactions between all faces of A and all vertices of B is given by a face-vertex interaction matrix:

$$
{}^{AB}\Phi^{fv} = [{}^{AB}\phi^{fv}_{ji}] = \begin{array}{c|cccc} & v_1^B & v_2^B & \cdots & v_{m_v^B}^B \\ \hline f_1^A & {}^{AB}\phi^{fv}_{1,1} & {}^{AB}\phi^{fv}_{1,2} & \cdots & {}^{AB}\phi^{fv}_{1,m_v^B} \\ f_2^A & {}^{AB}\phi^{fv}_{2,1} & {}^{AB}\phi^{fv}_{2,2} & \cdots & {}^{AB}\phi^{fv}_{2,m_v^B} \\ \vdots & \vdots & \vdots & & \vdots \\ f_{m_f^A}^A & {}^{AB}\phi^{fv}_{m_f^A,1} & {}^{AB}\phi^{fv}_{m_f^A,2} & \cdots & {}^{AB}\phi^{fv}_{m_f^A,m_v^B} \end{array}
$$

$$
{}^{AB}\phi^{fv}_{ji} = sign({}^{AB}h^{fv}_{ji}(R_1^0, r_1^0)) = sign(d_s^E(v_i^B, f_j^A))
$$

Feature Interactions (cont') - Edge-Edge Interaction

- The interactions between all edges of A and all edges of B is given by a edge-edge interaction matrix:

$$
{}^{AB}\Phi^{ee} = [{}^{AB}\phi^{ee}_{kl}] = \begin{array}{c|cccc}
 & e_1^B & e_2^B & \cdots & e^B_{m_e^B} \\
\hline
e_1^A & {}^{AB}\phi^{ee}_{1,1} & {}^{AB}\phi^{ee}_{1,2} & \cdots & {}^{AB}\phi^{ee}_{1,m_e^B} \\
e_2^A & {}^{AB}\phi^{ee}_{2,1} & {}^{AB}\phi^{ee}_{2,2} & \cdots & {}^{AB}\phi^{ee}_{2,m_e^B} \\
\vdots & & & & \\
e^A_{m_e^A} & {}^{AB}\phi^{ee}_{m_e^A,1} & {}^{AB}\phi^{ee}_{m_e^A,1} & \cdots & {}^{AB}\phi^{ee}_{m_e^A,m_e^B}
\end{array}
$$

$$
{}^{AB}\phi^{ee}_{kl} = sign\,({}^{AB}h^{ee}_{kl}(R_1^0, r_1^0)) = sign\,((\widetilde{u}^{\,0}_{e_k^A} u^0_{e_l^B})^t (r^0_{v^A_{k_1}, v^B_{l_1}}))
$$

Feature Interaction Matrix

- The convex decomposition of A and B are denoted as:

$$cv(A) = \{A_1, A_2, \ldots, A_{m_{cv}^A}\} \qquad cv(B) = \{B_1, B_2, \ldots, B_{m_{cv}^B}\}$$

- Feature Interaction Matrix:

$$^{AB}\Phi = \begin{bmatrix} ^{A_1B_1}\Phi & ^{A_1B_2}\Phi & \cdots & ^{A_1B_{m_{cv}^B}}\Phi \\ ^{A_2B_1}\Phi & ^{A_2B_2}\Phi & \cdots & ^{A_2B_{m_{cv}^B}}\Phi \\ \vdots & \vdots & & \vdots \\ ^{A_{m_{cv}^A}B_1}\Phi & ^{A_{m_{cv}^A}B_2}\Phi & \cdots & ^{A_{m_{cv}^A}B_{m_{cv}^B}}\Phi \end{bmatrix}$$

$$^{A_iB_j}\Phi = \begin{bmatrix} ^{A_iB_j}\Phi^{vf} & 0 & 0 \\ 0 & ^{A_iB_j}\Phi^{fv} & 0 \\ 0 & 0 & ^{A_iB_j}\Phi^{ee} \end{bmatrix}$$

Uniqueness: Many-to-One Mapping

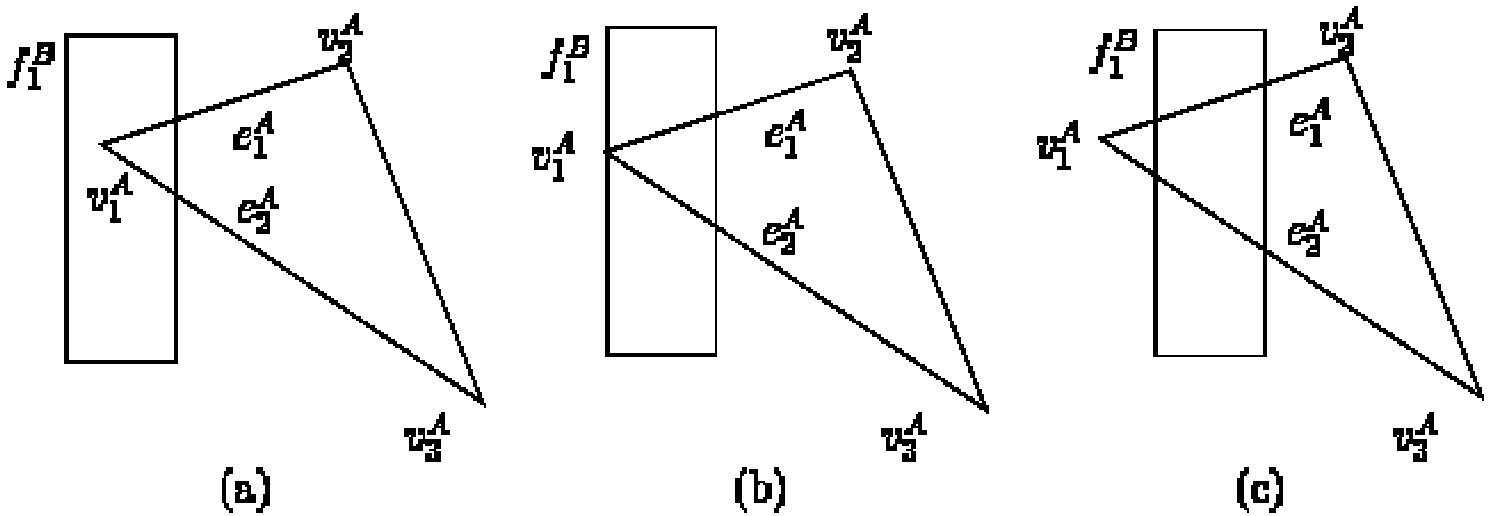
- Let C denote the configuration space of A.
- Let $\Gamma = \{ {}^{AB}\Phi \}$ denote the set of all feature interaction matrices.
- A configuration $q = (R_1^0, r_1^0) \in C$ of A corresponds to a feature interaction matrix ${}^{AB}\Phi \in \Gamma$.
- The mapping $g : C \to \Gamma$ is defined as: ${}^{AB}\Phi = g(q)$ where each element ${}^{AB}\Phi_{kl} = g_{kl}(q)$ is computed as:

$$g_{kl}(\mathbf{q}) = \begin{cases} {}^{AB}\phi_{kl}^{vf}(\mathbf{q}) & \text{if } {}^{AB}\phi_{kl} \text{ corresponds to a vertex-face interaction} \\ {}^{AB}\phi_{kl}^{fv}(\mathbf{q}) & \text{if } {}^{AB}\phi_{kl} \text{ corresponds to a face-vertex interaction} \\ {}^{AB}\phi_{kl}^{ee}(\mathbf{q}) & \text{if } {}^{AB}\phi_{kl} \text{ corresponds to an edge-edge interaction} \\ 0 & \text{otherwise} \end{cases}$$

Feasibility Verification of Hypothetical Feature Interaction Matrices

- Problem statement: Given a hypothetical, arbitrary feature interaction matrix, does there exist a configuration of the objects for which
  - the kinematic constraints imposed by the elements of the feature interaction matrix are satisfied
  - the parts do not penetrate?

Falsify Hypotheses - Penetration Information



Three different types of penetration exist. In Panel (a) vertex v_1^A is inside object B. In Panel (b) the objects penetrate each other while vertex v_1^A contacts face f_1^B. Panel (c) shows that the objects penetrate each other if an edge intersects with a face of the other object.

FIGURE 8.1. Three types of penetration of polyhedral objects.

Falsify Hypotheses - Penetration Information (cont’)

- Vertex inside object

  Objects A and B penetrate each other if one of the vertex-face interaction matrices contains a row with only –1 entries or one of the face-vertex interaction matrices contains a column with only –1 entries :

$$(\exists i)(\exists j)(\exists k)(\forall l)({}^{A_i B_j}\phi_{kl}^{vf} = -1)$$

$$(\exists i)(\exists j)(\exists l)(\forall k)({}^{A_i B_j}\phi_{kl}^{fv} = -1)$$

Falsify Hypotheses - Penetration Information (cont')

- Vertex contacting a face

  Objects A and B do penetrate each other if there exist i, j, k, l such that the following three conditions are satisfied:

$$^{A_i B_j}\phi_{kl}^{vf} = 0$$

$$(\forall f_m^{B_j} \in adj(f_l^{B_j}))(^{A_i B_j}\phi_{km}^{vf} = -1)$$

$$(\exists v_n^{A_i} \in cobound\ (v_k^{A_j}))(^{A_i B_j}\phi_{nl}^{vf} = -1)$$

Falsify Hypotheses
- Penetration Information (cont')

- Edge intersecting a face

  Let edge $e_k^{A_i}$ be bounded by vertices $v_{k_1}^{A_i}$ and $v_{k_2}^{A_i}$. Edge $e_k^{A_i}$ intersects face $f_l^{B_j}$ if the following two conditions are satisfied:

$$^{A_iB_j}\phi_{k_1 l}^{vf} \times {}^{A_iB_j}\phi_{k_2 l}^{vf} = -1$$

$$(\forall f_m^{B_j} \in adj(f_l^{B_j}))({}^{A_iB_j}\phi_{k_1 m}^{vf} = {}^{A_iB_j}\phi_{k_2 m}^{vf} = -1)$$

Falsify Hypotheses
- Optimization Based Method

$$\min \quad G = d_p^G(A, B)$$

$$subject \quad to:$$

$$^{AB}h_{rs}(R_1^0, r_1^0) = 0 \qquad \forall (r, s) \in ZEROS \; (^{AB}\Phi)$$

$$^{AB}h_{rs}(R_1^0, r_1^0) > 0 \qquad \forall (r, s) \in ONES \; (^{AB}\Phi)$$

$$^{AB}h_{rs}(R_1^0, r_1^0) < 0 \qquad \forall (r, s) \in MINUS - ONES \; (^{AB}\Phi)$$

Spatial Adjacency - Necessary Condition

- Let ${}^{AB}\Phi_i$ and ${}^{AB}\Phi_j$ be two spatially adjacent qualitative configurations and let matrix ${}^{AB}\Phi_i$ be more constrained than ${}^{AB}\Phi_j$. Only elements that are zero in ${}^{AB}\Phi_i$ can change during a transition from ${}^{AB}\Phi_i$ to ${}^{AB}\Phi_j$. Elements that are nonzero in ${}^{AB}\Phi_i$ can not change during the transition.

Spatial Adjacency
- Necessary and Sufficient Conditions

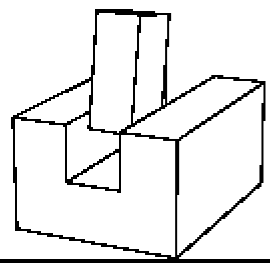
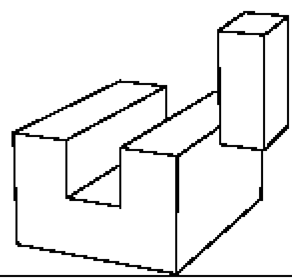
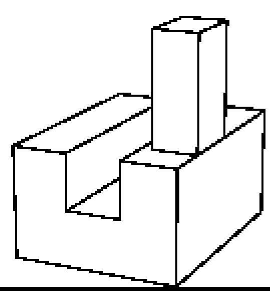
- Let ${}^{AB}\Phi_i$ be more constrained than ${}^{AB}\Phi_j$ and let and ${}^{AB}\Phi_i$ satisfy the necessary condition. Qualitative configurations ${}^{AB}\Phi_i$ and ${}^{AB}\Phi_j$ are adjacent at $q^* \in P_k({}^{AB}\Phi_i)$ if and only if there exists a twist t_1 such that:

$$({}^{AB}d_{mn})^t t_1 = 0 \qquad \forall (m,n) \in \left\{ (m,n) \middle| [{}^{AB}\Phi_j]_{mn} = 0 \right\}$$

$$({}^{AB}d_{mn})^t t_1 > 0 \qquad \forall (m,n) \in \left\{ (m,n) \middle| [{}^{AB}\Phi_i]_{mn} = 0 \wedge [{}^{AB}\Phi_j]_{mn} = +1 \right\}$$

$$({}^{AB}d_{mn})^t t_1 < 0 \qquad \forall (m,n) \in \left\{ (m,n) \middle| [{}^{AB}\Phi_i]_{mn} = 0 \wedge [{}^{AB}\Phi_j]_{mn} = -1 \right\}$$

Numerical Results for Verifying Spatial Adjacency

	Φ_i	Φ_j	Adjacent	CPU [ms]
a		Rotate 2.5° around e^1_z	yes	199.1
b		Rotate −1.3° around e^1_z	yes	91.8
c		Translate −0.4 along e^1_y and rotate 3.0° around e^1_z	no	1.7

Hypotheses Generation

- Hypotheses about less constrained neighbors of a configuration state ${}^{AB}\Phi_i$ can be generated by changing one or more zero elements of ${}^{AB}\Phi_j$ to ± 1 .
- Hypotheses about more constrained neighbors of a configuration state ${}^{AB}\Phi_i$ can be generated by changing one or more nonzero elements of ${}^{AB}\Phi_j$ to 0.
- Using local penetration information to rule out infeasible hypotheses in an early stage.

Contact Description

- A contact description between two non-convex polyhedra A and B is the union $\gamma^{A,B}$ of the contact descriptions between combinations of convex polyhedra A_i and B_j of the convex decompositions of A and B:

$$\gamma^{A,B} = \bigcup_{\substack{1 \le i \le m_{cv}^A \\ 1 \le j \le m_{cv}^B}} \gamma^{A_i, B_j}$$

Contact Information Extraction

- Vertex-face contacts

  Let A_i and B_j be convex polyhedra. There is contact between vertex $v_k^{A_i}$ and face $f_l^{B_j}$ if and only if

$$^{A_iB_j}\phi_{kl}^{vf} = 0$$

$$(\forall f_m^{B_j} \in adj(f_l^{B_j}))((^{A_iB_j}\phi_{km}^{vf} = -1) \vee (^{A_iB_j}\phi_{km}^{vf} = 0))$$

Contact Information Extraction (cont')

- Face-vertex contacts

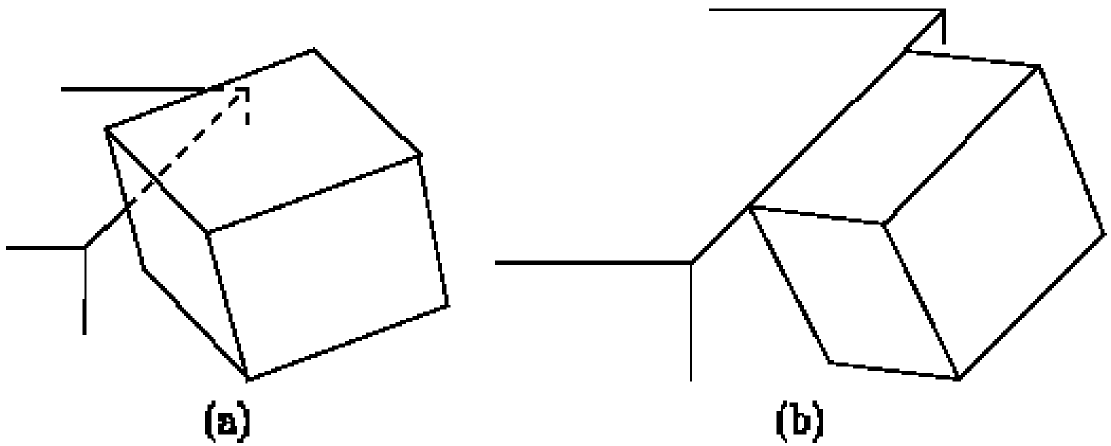
  Let A_i and B_j be convex polyhedra. There is contact between face $f_k^{A_i}$ and vertex $v_l^{B_j}$ if and only if

$$^{A_iB_j}\phi_{kl}^{fv} = 0$$

$$(\forall f_m^{A_i} \in adj(f_k^{A_i}))((^{A_iB_j}\phi_{km}^{fv} = -1) \vee (^{A_iB_j}\phi_{km}^{fv} = 0))$$

Contact Information Extraction (cont')

- Edge-edge contacts



Two types of edge-edge contacts are considered: *edge-edge-cross* contacts and *edge-edge-touch* contacts. An edge-edge-cross contact exist when two edges contact each other but are not parallel. This is shown in Frame (a). An edge-edge-touch contact exist when two edges contact each other while they are parallel. This is shown in Frame (b).

Contact Information Extraction (cont')

- Edge-edge-cross contact

  Let A_i and B_j be convex polyhedra. There exists and edge-edge cross contact between edge $e_k^{A_i}$ and $e_l^{B_j}$ if and only if the following three conditions are satisfied:

$$^{A_iB_j}\phi_{kl}^{ee} = 0$$

$$({}^{A_iB_j}\phi_{k_1l_1}^{vf} \neq {}^{A_iB_j}\phi_{k_2l_1}^{vf}) \vee ({}^{A_iB_j}\phi_{k_1l_2}^{vf} \neq {}^{A_iB_j}\phi_{k_2l_2}^{vf})$$

$$({}^{A_iB_j}\phi_{k_1l_1}^{fv} \neq {}^{A_iB_j}\phi_{k_1l_2}^{fv}) \vee ({}^{A_iB_j}\phi_{k_2l_1}^{fv} \neq {}^{A_iB_j}\phi_{k_2l_2}^{fv})$$

Contact Information Extraction (cont')

- Edge-edge-touch contact

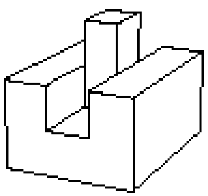
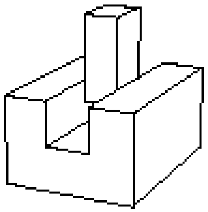
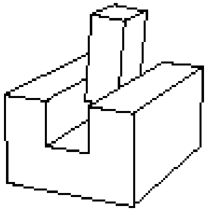
  Let A_i and B_j be convex polyhedra. There exists and edge-edge touch contact between edge $e_k^{A_i}$ and $e_l^{B_j}$ if and only if the following two conditions are satisfied:

$$({}^{A_iB_j}\phi^{vf}_{k_1l_1} = 0 \wedge {}^{A_iB_j}\phi^{vf}_{k_1l_2} = 0) \wedge ({}^{A_iB_j}\phi^{vf}_{k_2l_1} = 0 \wedge {}^{A_iB_j}\phi^{vf}_{k_2l_2} = 0)$$

$$({}^{A_iB_j}\phi^{fv}_{k_3l_1} = {}^{A_iB_j}\phi^{fv}_{k_4l_1} = -1) \vee ({}^{A_iB_j}\phi^{fv}_{k_3l_2} = {}^{A_iB_j}\phi^{fv}_{k_4l_2} = -1) \vee$$

$$({}^{A_iB_j}\phi^{vf}_{k_1l_3} = {}^{A_iB_j}\phi^{vf}_{k_1l_4} = -1) \vee ({}^{A_iB_j}\phi^{vf}_{k_2l_3} = {}^{A_iB_j}\phi^{vf}_{k_2l_4} = -1)$$

Numerical Results for Elementary Contact Extraction

	$\gamma^{A,B}$	CPU [ms]
	$v_0^{A_1} - f_{102}^{B_1}, v_1^{A_1} - f_{102}^{B_1}, v_2^{A_1} - f_{102}^{B_1}, v_3^{A_1} - f_{102}^{B_1}, v_0^{A_1} - f_1^{B_2}, v_1^{A_1} - f_1^{B_2}, v_0^{A_1} - f_{202}^{B_2}, v_1^{A_1} - f_{202}^{B_2}, e_0^{A_1} - e_6^{B_2}, e_0^{A_1} - e_9^{B_2}, e_1^{A_1} - e_6^{B_2}, e_1^{A_1} - e_9^{B_2}, e_4^{A_1} - e_9^{B_2}, e_5^{A_1} - e_9^{B_2}, e_7^{A_1} - e_9^{B_2}$	4.9
	$v_0^{A_1} - f_1^{B_2}, v_1^{A_1} - f_1^{B_2}, e_0^{A_1} - e_6^{B_2}, e_1^{A_1} - e_6^{B_2}$	1.0
	$v_0^{A_1} - f_1^{B_2}, v_1^{A_1} - f_1^{B_2}$	0.8

A Simple Metric for Planning

- Define the *cost-to-go* from ${}^{AB}\phi_i$ to ${}^{AB}\phi_{goal}$ along trajectory $T_{tr}({}^{AB}\phi_i, {}^{AB}\phi_{goal})$ as:

$$CTG(T_{tr}({}^{AB}\phi_i, {}^{AB}\phi_{goal})) = n - \sum_{k=0}^{n-1} \beta N_c({}^{AB}\phi_{i+k})$$

Automatic Synthesis of Qualitative Configuration Models and Discrete Event Controllers

- Synthesize-Discrete-Event-Controller algorithm integrates the methods for hypothesis generation, hypotheses testing and contact information extracting into one algorithm that synthesizes a discrete even controller.
- The search should be terminated if an acceptable path from the goal configuration to the start configuration has been found or if all states have been expanded.

Conclusions

- Two representations are considered: sets of elementary contacts and feature interaction matrices.
- A complete, informed generation of hypotheses algorithm has been developed.
- Computational methods have been developed to verify the geometrical feasibility of hypothetical contact and configuration descriptions.
- The spatial adjacency of two qualitative configurations can be verified by polar cone techniques.
- Contact information can be extracted from feature interaction matrices.

Future Work: Compliant, Fine Motion Planning

Related Works:

- LMT approach:
  - Computing motion strategies given bounded uncertainty using pre-images in configuration space (C-space) – by Lozano-Perez, Mason and Taylor.
  - Plan compliant motions in a finite state space for 3D translation without rotation – by S.J.Buckley.

Future Work: Compliant, Fine Motion Planning (cont')

- Two phase approach: first a nominal trajectory in free space is computed without considering uncertainty, uncertainties are taken into account in the second phase.
  - Design of nominal velocities and inverse damping matrices to guarantee force assembly – by Schimmels and Peshkin.
  - Let a deviation from the nominal trajectory trigger an error recovery strategy or patch plan – by Xiao and Volz.
  - Generating hypothetical contact formations – by Xiao and Zhang.
  - Linearizing the configuration space around critical points – by Dakin and Popplestone.