Synthesizing Discrete Event Controllers for Robotic Assembly by Automatic Construction of Qualitative Contact Models

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- Sets of elementary contacts
- Feature interaction matrices
- Conclusions
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Automatic Assembly

- Position-based control
  - Easy to program
  - Expensive for precision position control
- Compliant motion control
  - Reduce the uncertainty
  - Requires less accurate and less expensive equipment
  - Hard to program
Qualitative Models

- Qualitative state adjacency graph
  - Expression: \( G = (\Gamma, T) \)
  - \( \Gamma \): space of qualitative configuration representations
  - \( T \): set of arcs connecting adjacent configuration states

- Augmented qualitative state adjacency graph
  - Expression: \( G_a = (\Gamma, T, T_d) \)
  - \( T_d \): set of all desired transitions
Discrete Event Controller

- Discrete states: the qualitative distinct contact or configuration states
- Discrete events: perceived state transitions
Sets of Elementary Contacts – Contact Between Two Polyhedra

- A contact description is a set of elementary face-vertex, vertex-face and edge-edge contacts:

\[ \gamma_{A,B} = \{ f_i^A - v_j^B, \ldots, v_k^A - f_l^B, \ldots, e_m^A - e_n^B \} \]

The set of all syntactically correct contact description is denoted by \( \Gamma_{A,B} \).

- A qualitative contact state is a geometrically feasible contact description \( \gamma_{A,B} \). The set of all contact states is denoted by \( \Gamma^c_{A,B} \subset \Gamma_{A,B} \).
Problem statement: Given a hypothetical contact description, does there exist a configuration of the objects for which:

- the kinematic equality and bounding constraints imposed by the elementary contacts are satisfied.
- the parts do not penetrate?
Criteria for Geometric Feasibility – Kinematic Constraints and Bounds

- Vertex-face contact

\[
\begin{align*}
\mathbf{h}_{eq}^I (f_j^B, v_i^A) & \overset{\text{def}}{=} d_s^E (f_j^B, v_i^A) = 0 \\
\mathbf{h}_{bd}^I (f_j^B, v_i^A) & \overset{\text{def}}{=} \prod_{f_c^B \in \text{cv}(f_j^B)} \sum_{p_k^B \in \text{bp}(f_c^B)} \mu(-d_s^E (p_k^B, v_i^A)) = 0
\end{align*}
\]
Criteria for Geometric Feasibility
– Kinematic Constraints and Bounds (cont’)

➢ Face-vertex contact

\[
h_{\text{eq}}^E(v_i, f_j^A) = 0
\]

\[
h_{\text{bd}}^E(v_i, f_j^A) = 0
\]
Criteria for Geometric Feasibility – Kinematic Constraints and Bounds (cont’)

- **Edge-edge contact**

\[ h_{eq}^{III}(d_j^B, e_i^A) = (n^{0}_{e_i^A e_j^B})^t (r^{0}_{v_k^A, v_n^B}) = 0 \]

\[ h_{bd}^{III}(e_j^B, e_i^A) = \mu(\alpha_{e_j^B e_i^A}^B) + \mu(l_{e_j^B e_i^A}^B) + \mu(\alpha_{e_j^B e_i^A}^A) + \mu(l_{e_i^A e_j^B}^A) = 0 \]
Criteria for Geometric Feasibility – Nonpenetration Condition

\[ d_p^G (A, B) = \sum_{A_k \in cv(A)} \left[ \sum_{B_l \in cv(B)} d_p^G (A_k, B_l) \right] = 0 \]

\[ d_p^G (A_k, B_l) = \begin{cases} 
(\rho_A + \rho_B)(1 - g(A_k, B_l)) & \text{if } g(A_k, B_l) \leq 1 \\
0 & \text{if } g(A_k, B_l) > 1 
\end{cases} \]

\[ g(A_k, B_l) = \min \sigma \]

subject to \( A_k(\sigma) \cap B_l(\sigma) \neq 0 \)
Optimization Problem

$$\min \ G = d_p^G (A, B)$$

subject to:

$$\forall (v_i^A - f_j^B) \in \gamma^{A,B}$$

$$\forall (f_j^A - v_i^B) \in \gamma^{A,B}$$

$$\forall (e_i^A - e_j^B) \in \gamma^{A,B}$$

$$\begin{align*}
\{ h^{I}_{eq} (f_j^B, v_i^A) &= 0 \\
\{ h^{I}_{bd} (f_j^B, v_i^A) &= 0 \\
\{ h^{II}_{eq} (v_i^B, f_j^A) &= 0 \\
\{ h^{II}_{bd} (v_i^B, f_j^A) &= 0 \\
\{ h^{III}_{eq} (e_j^B, e_i^A) &= 0 \\
\{ h^{III}_{bd} (e_j^B, e_i^A) &= 0 
\end{align*}$$
Unconstrained Optimization Problem

\[ G = \left( d_p^G \right)^2 \]

\[ + \sum_{(v_i^A - f_j^B)} \left[ h_{eq}^I \left( f_j^B, v_i^A \right) \right]^2 + \sum_{(v_i^A - f_j^B)} h_{bd}^I \left( f_j^B, v_i^A \right) \]

\[ + \sum_{(f_j^A - v_i^B)} \left[ h_{eq}^II \left( v_i^B, f_j^A \right) \right]^2 + \sum_{(f_j^A - v_i^B)} h_{bd}^II \left( v_i^B, f_j^A \right) \]

\[ + \sum_{(e_i^A - e_j^B)} \left[ h_{eq}^III \left( e_j^B, e_i^A \right) \right]^2 + \sum_{(e_i^A - e_j^B)} h_{bd}^III \left( e_j^B, e_i^A \right) \]
Solution of Optimization Problem

- The optimization problem can be solved numerically by simulating a viscoelastic dynamic system.
- The cost function $G$ is a function of configuration of the mobile object, which is $(\dot{R}_1^0, \dot{r}_1^0)$.
- A dynamic state equation:
  $$
  \begin{bmatrix}
  \nu_1^0 \\
  w_1^0
  \end{bmatrix} = -B^{-1} \begin{bmatrix}
  f_G \\
  \tau_G
  \end{bmatrix}
  $$
- Gradient descent algorithms:
  $$
  \begin{cases}
  \dot{\dot{R}}_1^0 = \ddot{\omega}_1^0 R_1^0 \\
  \dot{\dot{r}}_1^0 = \nu_1^0
  \end{cases}
  $$
Table 2: Experimental results for the dovetail assembly and the hollebol piece.

<table>
<thead>
<tr>
<th></th>
<th>$\gamma_{A,B}$</th>
<th>$G$</th>
<th>$n_{ok}$</th>
<th>$n_{bad}$</th>
<th>result</th>
</tr>
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<tbody>
<tr>
<td>dovetail</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>$v_0^A - f_1^B$, $e_0^A - e_0^B$</td>
<td>0.0010</td>
<td>17</td>
<td>10</td>
<td>feasible</td>
</tr>
<tr>
<td>2</td>
<td>$v_2^A - f_3^B$, $e_0^A - e_0^B$</td>
<td>0.0012</td>
<td>6</td>
<td>0</td>
<td>feasible</td>
</tr>
<tr>
<td>3</td>
<td>$v_0^A - f_1^B$, $e_2^A - e_0^B$</td>
<td>3.8596</td>
<td>83</td>
<td>28</td>
<td>infeasible</td>
</tr>
<tr>
<td>hollebol piece</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>$v_0^A - f_1^B$, $e_0^A - e_0^B$</td>
<td>0.0002</td>
<td>8</td>
<td>1</td>
<td>feasible</td>
</tr>
<tr>
<td>2</td>
<td>$v_2^A - f_3^B$, $e_0^A - e_0^B$</td>
<td>0.0013</td>
<td>49</td>
<td>9</td>
<td>feasible</td>
</tr>
<tr>
<td>3</td>
<td>$f_1^A - v_0^A$, $f_{12} - v_{12}$, $f_{13} - v_{13}$, $f_{14} - v_{14}$, $v_0^A - f_1^B$, $v_2^A - f_3^B$, $v_0^A - f_1^B$, $v_2^A - f_3^B$</td>
<td>0.0006</td>
<td>13</td>
<td>4</td>
<td>feasible</td>
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<tr>
<td>4</td>
<td>$v_2^A - f_3^B$, $v_0^A - f_1^B$, $v_0^A - f_1^B$, $v_0^A - f_1^B$, $v_0^A - f_1^B$, $v_0^A - f_1^B$, $v_0^A - f_1^B$, $v_0^A - f_1^B$</td>
<td>6.0511</td>
<td>51</td>
<td>9</td>
<td>infeasible</td>
</tr>
</tbody>
</table>

Figure 8: Verification of contact hypotheses for two different object pairs. Panel (a) shows verification of geometrically feasible contact hypothesis $\gamma_{A,B} = \{v_0^A - f_1^B, e_0^A - e_0^B\}$ (identical to Figure 1) for the dovetail assembly. Intermediate configurations are shown to emphasise that verification is a dynamic process. Panels (b) and (c) show testing of contact hypotheses $\gamma_{A,B} = \{f_1^A - v_0^B, f_{12} - v_{12}, f_{13} - v_{13}, f_{14} - v_{14}, v_0^A - f_1^B, v_2^A - f_3^B\}$ and $\gamma_{A,B} = \{f_{12} - v_{12}, f_{13} - v_{13}, f_{14} - v_{14}, f_1^A - v_0^B, f_{14} - v_1^B\}$ for the "hollebol" piece, respectively. For clarity, only the initial and final configuration are shown.
Discussion about Representing contact using Sets of Elementary Contacts

- **Major problem:** the lack of efficient methods to verify spatial adjacency of two states.
- **Solution:** search for a representation for which adjacency can be verified easily – a representation using feature interaction matrices.
Feature Interactions - Vertex-Face Interaction

- The interactions between all vertices of A and all faces of B is given by a vertex-face interaction matrix:

\[
AB \phi_{vf} = [AB \phi_{ij}^{vf}] =
\begin{bmatrix}
\begin{array}{cccc}
\phi_{v_1}^{f_1} & \phi_{v_1}^{f_2} & \cdots & \phi_{v_1}^{f_m} \\
\phi_{v_2}^{f_1} & \phi_{v_2}^{f_2} & \cdots & \phi_{v_2}^{f_m} \\
\cdots & \cdots & \cdots & \cdots \\
\phi_{v_n}^{f_1} & \phi_{v_n}^{f_2} & \cdots & \phi_{v_n}^{f_m}
\end{array}
\end{bmatrix}
\]

\[
AB \phi_{ij}^{vf} = \text{sign} (AB h_{ij}^{vf} (R_1^0, r_1^0)) = \text{sign} (d_s^E (f_j^B, v_i^A))
\]
Feature Interactions (cont’)
- Face-Vertex Interaction

The interactions between all faces of A and all vertices of B is given by a face-vertex interaction matrix:

\[
A^B \Phi_{f^v} = \begin{bmatrix} A^B \phi_{j_i}^{f^v} \\ \vdots \end{bmatrix} = \begin{bmatrix} v_1^B & v_2^B & \cdots & v_{n_2}^B \\ \vdots & \vdots & \ddots & \vdots \\ v_1^A & v_2^A & \cdots & v_{n_2}^A \\ \vdots & \vdots & \ddots & \vdots \\ v_{m_1}^A & v_{m_2}^A & \cdots & v_{m_2}^A \end{bmatrix}
\]

\[
A^B \phi_{j_i}^{f^v} = \text{sign} \left( A^B h_{j_i}^{f^v} \left( R_1^0, r_1^0 \right) \right) = \text{sign} \left( d_s^E \left( v_i^B, f_j^A \right) \right)
\]
Feature Interactions (cont’)

- Edge-Edge Interaction

The interactions between all edges of A and all edges of B is given by an edge-edge interaction matrix:

\[
\begin{align*}
\Phi_{ee} & = \begin{bmatrix} \phi_{ee}^{AB} \end{bmatrix} = \\
& = \begin{bmatrix}
\phi_{11}^{AB} & \phi_{12}^{AB} & \ldots & \phi_{1n}^{AB} \\
\phi_{21}^{AB} & \phi_{22}^{AB} & \ldots & \phi_{2n}^{AB} \\
\vdots & \vdots & \ddots & \vdots \\
\phi_{m1}^{AB} & \phi_{m2}^{AB} & \ldots & \phi_{mn}^{AB}
\end{bmatrix}
\end{align*}
\]

\[
\phi_{ee}^{AB} = \text{sign} \left( h_{kl}^{ee} \left( R_{1}^{0}, r_{1}^{0} \right) \right) = \text{sign} \left( \left( \tilde{u}_{e_{k}}^{0} u_{e_{l}}^{0} \right)^{t} \left( r_{v_{k_{1}}, v_{l_{1}}}^{0} \right) \right)
\]
Feature Interaction Matrix

- The convex decomposition of A and B are denoted as:
  \[ cv(A) = \{ A_1, A_2, \ldots, A_{m_cv} \} \quad cv(B) = \{ B_1, B_2, \ldots, B_{m_cv} \} \]

- Feature Interaction Matrix:

  \[
  \begin{align*}
  A_{B \Phi} &= \begin{bmatrix}
  A_1B_1 \Phi & A_1B_2 \Phi & \cdots & A_1B_{m_cv} \Phi \\
  A_2B_1 \Phi & A_2B_2 \Phi & \cdots & A_2B_{m_cv} \Phi \\
  \vdots & \vdots & \ddots & \vdots \\
  A_{m_cv}B_1 \Phi & A_{m_cv}B_2 \Phi & \cdots & A_{m_cv}B_{m_cv} \Phi 
  \end{bmatrix} \\
  A_{Bj} \Phi &= \begin{bmatrix}
  A_{Bj} \Phi^{vf} & 0 & 0 \\
  0 & A_{Bj} \Phi^{fv} & 0 \\
  0 & 0 & A_{Bj} \Phi^{ee}
  \end{bmatrix}
  \end{align*}
  \]
Uniqueness: Many-to-One Mapping

- Let $C$ denote the configuration space of $A$.
- Let $\Gamma = \{^{AB}\Phi\}$ denote the set of all feature interaction matrices.
- A configuration $q = (R_1^0, r_1^0) \in C$ of $A$ corresponds to a feature interaction matrix $^{AB}\Phi \in \Gamma$.
- The mapping $g : C \rightarrow \Gamma$ is defined as: $^{AB}\Phi = g(q)$ where each element $^{AB}\Phi_{kl} = g_{kl}(q)$ is computed as:

$$g_{kl}(q) = \begin{cases} ^{AB}\phi_{v_f}^f(q) & \text{if } ^{AB}\phi_{v_f}^f \text{ corresponds to a vertex-face interaction} \\ ^{AB}\phi_{f_v}^f(q) & \text{if } ^{AB}\phi_{f_v}^f \text{ corresponds to a face-vertex interaction} \\ ^{AB}\phi_{e_e}^e(q) & \text{if } ^{AB}\phi_{e_e}^e \text{ corresponds to an edge-edge interaction} \\ 0 & \text{otherwise} \end{cases}$$
Feasibility Verification of Hypothetical Feature Interaction Matrices

- Problem statement: Given a hypothetical, arbitrary feature interaction matrix, does there exist a configuration of the objects for which
  - the kinematic constraints imposed by the elements of the feature interaction matrix are satisfied
  - the parts do not penetrate?
Three different types of penetration exist. In Panel (a) vertex $v_1^A$ is inside object $B$. In Panel (b) the objects penetrate each other while vertex $v_1^A$ contacts face $f_1^B$. Panel (c) shows that the objects penetrate each other if an edge intersects with a face of the other object.

**Figure 8.1.** Three types of penetration of polyhedral objects.
Falsify Hypotheses
- Penetration Information (cont’)

- Vertex inside object

Objects A and B penetrate each other if one of the vertex-face interaction matrices contains a row with only \(-1\) entries or one of the face-vertex interaction matrices contains a column with only \(-1\) entries:

$$(\exists i)(\exists j)(\exists k)(\forall l)(A_{iB_j} \phi_{vl}^{vf} = -1)$$

$$(\exists i)(\exists j)(\exists l)(\forall k)(A_{iB_j} \phi_{kl}^{fv} = -1)$$
Falsify Hypotheses
- Penetration Information (cont’)

- Vertex contacting a face

Objects A and B do penetrate each other if there exist i, j, k, l such that the following three conditions are satisfied:

\[ A_i B_j \phi_{kl}^{vf} = 0 \]

\[ (\forall f_m^B \in \text{adj} (f_l^B )) (A_i B_j \phi_{km}^{vf} = -1) \]

\[ (\exists v_n^A \in \text{cobound} (v_k^A )) (A_i B_j \phi_{nl}^{vf} = -1) \]
Falsify Hypotheses
- Penetration Information (cont’)

- Edge intersecting a face
  - Let edge $e_{k}^{A_i}$ be bounded by vertices $v_{k_1}^{A_i}$ and $v_{k_2}^{A_i}$. Edge $e_{k}^{A_i}$ intersects face $f_{l}^{B_j}$ if the following two conditions are satisfied:

\[ A_{i}B_{j} \phi_{k_1l}^{vf} \times A_{i}B_{j} \phi_{k_2l}^{vf} = -1 \]

\[ (\forall f_{m}^{B_j} \in \text{adj} (f_{l}^{B_j})) (A_{i}B_{j} \phi_{k_1m}^{vf} = A_{i}B_{j} \phi_{k_2m}^{vf} = -1) \]
Falsify Hypotheses
- Optimization Based Method

\[
\begin{align*}
\min & \quad G = d_p^G (A, B) \\
\text{subject to :} & \\
AB & h_{rs} (R_1^0, r_1^0) = 0 \quad \forall (r, s) \in ZEROS \quad (AB \Phi) \\
AB & h_{rs} (R_1^0, r_1^0) > 0 \quad \forall (r, s) \in ONES \quad (AB \Phi) \\
AB & h_{rs} (R_1^0, r_1^0) < 0 \quad \forall (r, s) \in MINUS - ONES \quad (AB \Phi)
\end{align*}
\]
Spatial Adjacency
- Necessary Condition

Let $^A B \Phi_i$ and $^A B \Phi_j$ be two spatially adjacent qualitative configurations and let matrix $^A B \Phi_i$ be more constrained than $^A B \Phi_j$. Only elements that are zero in $^A B \Phi_i$ can change during a transition from $^A B \Phi_i$ to $^A B \Phi_j$. Elements that are nonzero in $^A B \Phi_i$ can not change during the transition.
Spatial Adjacency
- Necessary and Sufficient Conditions

Let $^{AB} \Phi_i$ be more constrained than $^{AB} \Phi_j$ and let $^{AB} \Phi_i$ and $^{AB} \Phi_j$ satisfy the necessary condition. Qualitative configurations $^{AB} \Phi_i$ and $^{AB} \Phi_j$ are adjacent at $q^* \in P_k(^{AB} \Phi_i)$ if and only if there exists a twist $t_1$ such that:

$^{AB} d_{mn} t_1 = 0 \quad \forall (m,n) \in \{(m,n)|[^{AB} \Phi_j]_{mn} = 0\}$

$^{AB} d_{mn} t_1 > 0 \quad \forall (m,n) \in \{(m,n)|[^{AB} \Phi_i]_{mn} = 0 \land[^{AB} \Phi_j]_{mn} = +1\}$

$^{AB} d_{mn} t_1 < 0 \quad \forall (m,n) \in \{(m,n)|[^{AB} \Phi_i]_{mn} = 0 \land[^{AB} \Phi_j]_{mn} = -1\}$
Numerical Results for Verifying Spatial Adjacency

<table>
<thead>
<tr>
<th></th>
<th>$\Phi_i$</th>
<th>$\Phi_j$</th>
<th>Adjacent</th>
<th>CPU [ms]</th>
</tr>
</thead>
<tbody>
<tr>
<td>a</td>
<td></td>
<td>Rotate 2.5° around $e_z^1$</td>
<td>yes</td>
<td>199.1</td>
</tr>
<tr>
<td>b</td>
<td></td>
<td>Rotate -1.3° around $e_z^2$</td>
<td>yes</td>
<td>91.8</td>
</tr>
<tr>
<td>c</td>
<td></td>
<td>Translate -0.4 along $e_y^1$ and rotate 3.0° around $e_z^1$</td>
<td>no</td>
<td>1.7</td>
</tr>
</tbody>
</table>
Hypotheses Generation

- Hypotheses about less constrained neighbors of a configuration state $^{AB} \Phi_i$ can be generated by changing one or more zero elements of $^{AB} \Phi_j$ to $\pm 1$.

- Hypotheses about more constrained neighbors of a configuration state $^{AB} \Phi_i$ can be generated by changing one or more nonzero elements of $^{AB} \Phi_j$ to 0.

- Using local penetration information to rule out infeasible hypotheses in an early stage.
A contact description between two non-convex polyhedra $A$ and $B$ is the union of the contact descriptions between combinations of convex polyhedra $A_i$ and $B_j$ of the convex decompositions of $A$ and $B$:

$$\gamma^{A,B} = \bigcup_{1 \leq i \leq m^A_{cv}, 1 \leq j \leq m^B_{cv}} \gamma^{A_i,B_j}$$
Vertex-face contacts

Let $A_i$ and $B_j$ be convex polyhedra. There is contact between vertex $v^A_i$ and face $f^B_j$ if and only if

$$A_iB_j \phi_{kl}^{vf} = 0$$

$$(\forall f^B_j \in \text{adj}(f^B_j))(A_iB_j \phi_{km}^{vf} = -1) \lor (A_iB_j \phi_{km}^{vf} = 0)$$
Face-vertex contacts

Let $A_i$ and $B_j$ be convex polyhedra. There is contact between face $f_k^{A_i}$ and vertex $v_l^{B_j}$ if and only if

$$A_iB_j \phi_{f_{kl}}^{f_{y}} = 0$$

$$(\forall f_m^{A_i} \in \text{adj}(f_k^{A_i}))((A_iB_j \phi_{km}^{f_{y}} = -1) \lor (A_iB_j \phi_{km}^{f_{y}} = 0))$$
Contact Information Extraction (cont’)

- Edge-edge contacts

Two types of edge-edge contacts are considered: edge-edge-cross contacts and edge-edge-touch contacts. An edge-edge-cross contact exist when two edges contact each other but are not parallel. This is shown in Frame (a). An edge-edge-touch contact exist when two edges contact each other while they are parallel. This is shown in Frame (b).
Edge-edge-cross contact

Let $A_i$ and $B_j$ be convex polyhedra. There exists and edge-edge cross contact between edge $e_k^A_i$ and $e_l^B_j$ if and only if the following three conditions are satisfied:

$$A_i B_j \phi_{ee}^{kl} = 0$$

$$\left( A_i B_j \phi_{k_1 l_1}^{vf} \neq A_i B_j \phi_{k_2 l_1}^{vf} \right) \lor \left( A_i B_j \phi_{k_1 l_2}^{vf} \neq A_i B_j \phi_{k_2 l_2}^{vf} \right)$$

$$\left( A_i B_j \phi_{k_1 l_1}^{fv} \neq A_i B_j \phi_{k_1 l_2}^{fv} \right) \lor \left( A_i B_j \phi_{k_2 l_1}^{fv} \neq A_i B_j \phi_{k_2 l_2}^{fv} \right)$$
Edge-edge-touch contact

Let $A_i$ and $B_j$ be convex polyhedra. There exists and edge-edge touch contact between edge $e_k^{A_i}$ and $e_l^{B_j}$ if and only if the following two conditions are satisfied:

\[
\begin{aligned}
(A_i B_j \phi_{k_1 l_1}^{vf} = 0 \land A_i B_j \phi_{k_1 l_2}^{vf} = 0) \land (A_i B_j \phi_{k_2 l_1}^{vf} = 0 \land A_i B_j \phi_{k_2 l_2}^{vf} = 0) \\
(A_i B_j \phi_{k_3 l_1}^{fv} = A_i B_j \phi_{k_4 l_1}^{fv} = -1) \lor (A_i B_j \phi_{k_3 l_2}^{fv} = A_i B_j \phi_{k_4 l_2}^{fv} = -1) \lor \\
(A_i B_j \phi_{k_1 l_3}^{vf} = A_i B_j \phi_{k_1 l_4}^{vf} = -1) \lor (A_i B_j \phi_{k_2 l_3}^{vf} = A_i B_j \phi_{k_2 l_4}^{vf} = -1)
\end{aligned}
\]
Numerical Results for Elementary Contact Extraction

<table>
<thead>
<tr>
<th>$\gamma^{A,B}$</th>
<th>CPU [ms]</th>
</tr>
</thead>
<tbody>
<tr>
<td>$v_0^{A_1} - f_{1B_1}, v_1^{A_1} - f_{1B_1}, v_2^{A_1} - f_{1B_1}, v_3^{A_1} - f_{1B_1}, v_4^{A_1} - f_{1B_1}$</td>
<td>4.9</td>
</tr>
<tr>
<td>$v_0^{A_1} - f_{1B_1}, v_1^{A_1} - f_{1B_1}, v_2^{A_1} - f_{1B_1}, v_3^{A_1} - f_{1B_1}, v_4^{A_1} - f_{1B_1}$</td>
<td>1.0</td>
</tr>
<tr>
<td>$v_0^{A_1} - f_{1B_1}, v_1^{A_1} - f_{1B_1}, v_2^{A_1} - f_{1B_1}, v_3^{A_1} - f_{1B_1}, v_4^{A_1} - f_{1B_1}$</td>
<td>0.8</td>
</tr>
</tbody>
</table>
Define the \textit{cost-to-go} from $AB \phi_i$ to $AB \phi_{goal}$ along trajectory $T_{tr}(AB \phi_i, AB \phi_{goal})$ as:

$$CTG(T_{tr}(AB \phi_i, AB \phi_{goal})) = n - \sum_{k=0}^{n-1} \beta N_c(AB \phi_{i+k})$$
Automatic Synthesis of Qualitative Configuration Models and Discrete Event Controllers

- Synthesize-Discrete-Event-Controller algorithm integrates the methods for hypothesis generation, hypotheses testing and contact information extracting into one algorithm that synthesizes a discrete even controller.

- The search should be terminated if an acceptable path from the goal configuration to the start configuration has been found or if all states have been expanded.
Conclusions

- Two representations are considered: sets of elementary contacts and feature interaction matrices.
- A complete, informed generation of hypotheses algorithm has been developed.
- Computational methods have been developed to verify the geometrical feasibility of hypothetical contact and configuration descriptions.
- The spatial adjacency of two qualitative configurations can be verified by polar cone techniques.
- Contact information can be extracted from feature interaction matrices.
Future Work: Compliant, Fine Motion Planning

Related Works:

- **LMT approach:**
  - Computing motion strategies given bounded uncertainty using pre-images in configuration space (C-space) – by Lozano-Perez, Mason and Taylor.
  - Plan compliant motions in a finite state space for 3D translation without rotation – by S.J. Buckley.
Future Work: Compliant, Fine Motion Planning (cont’)

- Two phase approach: first a nominal trajectory in free space is computed without considering uncertainty, uncertainties are taken into account in the second phase.
- Design of nominal velocities and inverse damping matrices to guarantee force assembly – by Schimmels and Peshkin.
- Let a deviation from the nominal trajectory trigger an error recovery strategy or patch plan – by Xiao and Volz.
- Generating hypothetical contact formations – by Xiao and Zhang.
- Linearizing the configuration space around critical points – by Dakin and Popplestone.