Robotic Assembly by Qualitative Contact Models

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10/24/2003

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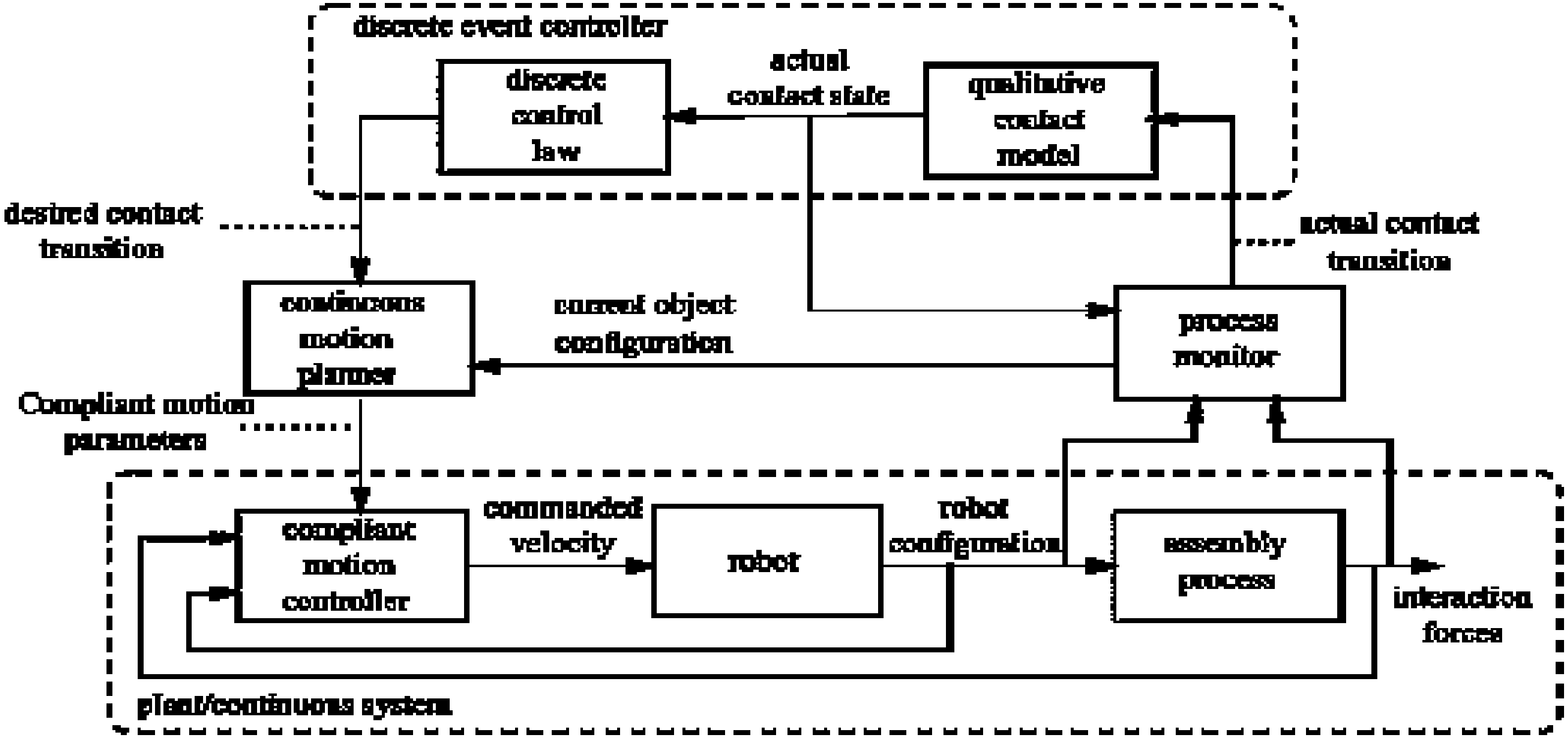
Automatic Assembly

- Position-based control
  - Easy to program
  - Expensive for precision position control
- Compliant motion control
  - Reduce the uncertainty
  - Requires less accurate and less expensive equipment
  - Hard to program

Automatic Assembly System



Discrete Event Controller

- Representation
- Hypotheses generation
- Hypotheses verification
- Spatial adjacency
- Contact information extraction
- Planning

Qualitative Models

- Qualitative state adjacency graph
  - Expression: $G = (\Gamma, T)$
  - Γ : space of qualitative configuration representations
  - T : set of arcs connecting adjacent configuration states
- Augmented qualitative state adjacency graph
  - Expression: $G_a = (\Gamma, T, T_d)$
  - T_d : set of all desired transitions

Sets of Elementary Contacts – Contact Between Two Polyhedra

- A contact description is a set of elementary face-vertex, vertex-face and edge-edge contacts:

$$\gamma_{A,B} = \{f_i^A - v_j^B, \ldots, v_k^A - f_l^B, \ldots, e_m^A - e_n^B\}$$

  The set of all syntactically correct contact description is denoted by $\Gamma_{A,B}$.
- A qualitative contact state is a geometrically feasible contact description $\gamma_{A,B}$. The set of all contact states is denoted by $\Gamma_{A,B}^c \subset \Gamma_{A,B}$.

Feature Interaction Matrix

- The convex decomposition of A and B are denoted as:

$$cv(A) = \{A_1, A_2, \ldots, A_{m_{cv}^A}\} \qquad cv(B) = \{B_1, B_2, \ldots, B_{m_{cv}^B}\}$$

- Feature Interaction Matrix:

$$ {}^{AB}\Phi = \begin{bmatrix} {}^{A_1B_1}\Phi & {}^{A_1B_2}\Phi & \cdots & {}^{A_1B_{m_{cv}^B}}\Phi \\ {}^{A_2B_1}\Phi & {}^{A_2B_2}\Phi & \cdots & {}^{A_2B_{m_{cv}^B}}\Phi \\ \vdots & \vdots & & \vdots \\ {}^{A_{m_{cv}^A}B_1}\Phi & {}^{A_{m_{cv}^A}B_2}\Phi & \cdots & {}^{A_{m_{cv}^A}B_{m_{cv}^B}}\Phi \end{bmatrix} \qquad {}^{A_iB_j}\Phi = \begin{bmatrix} {}^{A_iB_j}\Phi^{vf} & 0 & 0 \\ 0 & {}^{A_iB_j}\Phi^{fv} & 0 \\ 0 & 0 & {}^{A_iB_j}\Phi^{ee} \end{bmatrix} $$

Feature Interactions
- Vertex-Face Interaction

- The interactions between all vertices of A and all faces of B is given by a vertex-face interaction matrix:

$$
{}^{AB}\Phi^{vf} = [{}^{AB}\phi^{vf}_{ij}] = \begin{array}{c|cccc}
 & f_1^B & f_2^B & \cdots & f_{m_f^B}^B \\
\hline
v_1^A & {}^{AB}\phi^{vf}_{1,1} & {}^{AB}\phi^{vf}_{1,2} & \cdots & {}^{AB}\phi^{vf}_{1,m_f^B} \\
v_2^A & {}^{AB}\phi^{vf}_{2,1} & {}^{AB}\phi^{vf}_{2,2} & \cdots & {}^{AB}\phi^{vf}_{2,m_f^B} \\
\vdots & \vdots & \vdots & & \vdots \\
v_{m_v^A}^A & {}^{AB}\phi^{vf}_{m_v^A,1} & {}^{AB}\phi^{vf}_{m_v^A,2} & \cdots & {}^{AB}\phi^{vf}_{m_v^A,m_f^B}
\end{array}
$$

$$
{}^{AB}\phi^{vf}_{ij} = sign({}^{AB}h^{vf}_{ij}(R_1^0, r_1^0)) = sign(d_s^E(f_j^B, v_i^A))
$$

Feature Interactions (cont')
- Face-Vertex Interaction

- The interactions between all faces of A and all vertices of B is given by a face-vertex interaction matrix:

$$
{}^{AB}\Phi^{fv} = [{}^{AB}\phi^{fv}_{ji}] =
\begin{array}{c|cccc}
 & v_1^B & v_2^B & \cdots & v_{m_v^B}^B \\
\hline
f_1^A & {}^{AB}\phi^{fv}_{1,1} & {}^{AB}\phi^{fv}_{1,2} & \cdots & {}^{AB}\phi^{fv}_{1,m_v^B} \\
f_2^A & {}^{AB}\phi^{fv}_{2,1} & {}^{AB}\phi^{fv}_{2,2} & \cdots & {}^{AB}\phi^{fv}_{2,m_v^B} \\
\vdots & \vdots & \vdots & & \vdots \\
f_{m_f^A}^A & {}^{AB}\phi^{fv}_{m_f^A,1} & {}^{AB}\phi^{fv}_{m_f^A,2} & \cdots & {}^{AB}\phi^{fv}_{m_f^A,m_v^B}
\end{array}
$$

$$
{}^{AB}\phi^{fv}_{ji} = sign({}^{AB}h^{fv}_{ji}(R_1^0, r_1^0)) = sign(d_s^E(v_i^B, f_j^A))
$$

Feature Interactions (cont') - Edge-Edge Interaction

- The interactions between all edges of A and all edges of B is given by a edge-edge interaction matrix:

$$
{}^{AB}\Phi^{ee} = [{}^{AB}\phi^{ee}_{kl}] =
\begin{array}{c|cccc}
 & e_1^B & e_2^B & \cdots & e_{m_e^B}^B \\
\hline
e_1^A & {}^{AB}\phi^{ee}_{1,1} & {}^{AB}\phi^{ee}_{1,2} & \cdots & {}^{AB}\phi^{ee}_{1,m_e^B} \\
e_2^A & {}^{AB}\phi^{ee}_{2,1} & {}^{AB}\phi^{ee}_{2,2} & \cdots & {}^{AB}\phi^{ee}_{2,m_e^B} \\
\vdots & & & & \\
e_{m_e^A}^A & {}^{AB}\phi^{ee}_{m_e^A,1} & {}^{AB}\phi^{ee}_{m_e^A,1} & \cdots & {}^{AB}\phi^{ee}_{m_e^A,m_e^B}
\end{array}
$$

$$
{}^{AB}\phi^{ee}_{kl} = sign\,({}^{AB}h^{ee}_{kl}\,(R_1^0, r_1^0)) = sign\,((\tilde{u}^0_{e_k^A}\, u^0_{e_l^B})^t\,(r^0_{v_{k_1}^A, v_{l_1}^B}))
$$

Uniqueness: Many-to-One Mapping

- Let C denote the configuration space of A.
- Let $\Gamma = \{ {}^{AB}\Phi \}$ denote the set of all feature interaction matrices.
- A configuration $q = (R_1^0, r_1^0) \in C$ of A corresponds to a feature interaction matrix ${}^{AB}\Phi \in \Gamma$.
- The mapping $g : C \rightarrow \Gamma$ is defined as: ${}^{AB}\Phi = g(q)$ where each element ${}^{AB}\Phi_{kl} = g_{kl}(q)$ is computed as:

$$g_{kl}(\mathbf{q}) = \begin{cases} {}^{AB}\phi_{kl}^{vf}(\mathbf{q}) & \textit{if } {}^{AB}\phi_{kl} \textit{ corresponds to a vertex-face interaction} \\ {}^{AB}\phi_{kl}^{fv}(\mathbf{q}) & \textit{if } {}^{AB}\phi_{kl} \textit{ corresponds to a face-vertex interaction} \\ {}^{AB}\phi_{kl}^{ee}(\mathbf{q}) & \textit{if } {}^{AB}\phi_{kl} \textit{ corresponds to an edge-edge interaction} \\ 0 & \textit{otherwise} \end{cases}$$

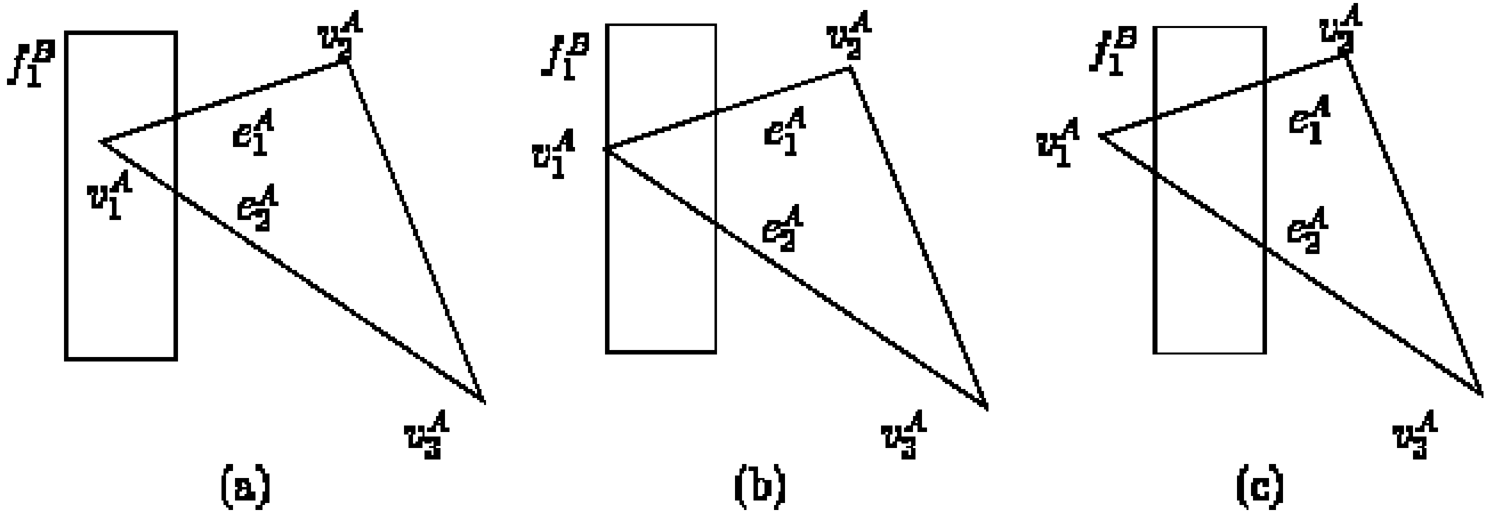
Hypotheses Generation

- Hypotheses about less constrained neighbors of a configuration state ${}^{AB}\Phi_i$ can be generated by changing one or more zero elements of ${}^{AB}\Phi_j$ to ± 1 .
- Hypotheses about more constrained neighbors of a configuration state ${}^{AB}\Phi_i$ can be generated by changing one or more nonzero elements of ${}^{AB}\Phi_j$ to 0.
- Using local penetration information to rule out infeasible hypotheses in an early stage.

Hypotheses Verification

- Problem statement: Given a hypothetical, arbitrary feature interaction matrix, does there exist a configuration of the objects for which
  - the kinematic constraints imposed by the elements of the feature interaction matrix are satisfied
  - the parts do not penetrate?

Falsify Hypotheses
- Penetration Information



Three different types of penetration exist. In Panel (a) vertex v_1^A is inside object B. In Panel (b) the objects penetrate each other while vertex v_1^A contacts face f_1^B. Panel (c) shows that the objects penetrate each other if an edge intersects with a face of the other object.

FIGURE 8.1. Three types of penetration of polyhedral objects.

Falsify Hypotheses - Penetration Information (cont')

- Vertex inside object

  Objects A and B penetrate each other if one of the vertex-face interaction matrices contains a row with only –1 entries or one of the face-vertex interaction matrices contains a column with only –1 entries :

$$(\exists i)(\exists j)(\exists k)(\forall l)({}^{A_i B_j}\phi_{kl}^{vf} = -1)$$

$$(\exists i)(\exists j)(\exists l)(\forall k)({}^{A_i B_j}\phi_{kl}^{fv} = -1)$$

Falsify Hypotheses - Penetration Information (cont')

- Vertex contacting a face

  Objects A and B do penetrate each other if there exist i, j, k, l such that the following three conditions are satisfied:

$$ {}^{A_i B_j}\phi_{kl}^{vf} = 0 $$

$$ (\forall f_m^{B_j} \in adj(f_l^{B_j}))({}^{A_i B_j}\phi_{km}^{vf} = -1) $$

$$ (\exists v_n^{A_i} \in cobound\ (v_k^{A_j}))({}^{A_i B_j}\phi_{nl}^{vf} = -1) $$

Falsify Hypotheses - Penetration Information (cont')

- Edge intersecting a face

  Let edge $e_k^{A_i}$ be bounded by vertices $v_{k_1}^{A_i}$ and $v_{k_2}^{A_i}$. Edge $e_k^{A_i}$ intersects face $f_l^{B_j}$ if the following two conditions are satisfied:

$$ {}^{A_i B_j}\phi_{k_1 l}^{vf} \times {}^{A_i B_j}\phi_{k_2 l}^{vf} = -1 $$

$$ (\forall f_m^{B_j} \in adj(f_l^{B_j}))({}^{A_i B_j}\phi_{k_1 m}^{vf} = {}^{A_i B_j}\phi_{k_2 m}^{vf} = -1) $$

Criteria for Geometric Feasibility – Non-penetration Condition

$$g(A_k, B_l) = \min \sigma$$

$$\text{subject to} \quad A_k(\sigma) \bigcap_{\sigma \in R^+} B_l(\sigma) \neq 0$$

$$d_p^G(A_k, B_l) = \begin{cases} (\rho_A + \rho_B)(1 - g(A_k, B_l)) & g(A_k, B_l) \leq 1 \\ 0 & g(A_k, B_l) > 1 \end{cases}$$

$$d_p^G(A, B) = \sum_{A_k \in cv(A)} [\sum_{B_l \in cv(B)} d_p^G(A_k, B_l)] = 0$$

Falsify Hypotheses
- Optimization Based Method

$$\min \quad G = d_p^G(A, B)$$

$$subject \quad to:$$

$$ {}^{AB}h_{rs}(R_1^0, r_1^0) = 0 \qquad \forall (r, s) \in ZEROS \quad ({}^{AB}\Phi)$$

$$ {}^{AB}h_{rs}(R_1^0, r_1^0) > 0 \qquad \forall (r, s) \in ONES \quad ({}^{AB}\Phi)$$

$$ {}^{AB}h_{rs}(R_1^0, r_1^0) < 0 \qquad \forall (r, s) \in MINUS - ONES \quad ({}^{AB}\Phi)$$

Spatial Adjacency - Necessary Condition

- Let ${}^{AB}\Phi_i$ and ${}^{AB}\Phi_j$ be two spatially adjacent qualitative configurations and let matrix ${}^{AB}\Phi_i$ be more constrained than ${}^{AB}\Phi_j$. Only elements that are zero in ${}^{AB}\Phi_i$ can change during a transition from ${}^{AB}\Phi_i$ to ${}^{AB}\Phi_j$. Elements that are nonzero in ${}^{AB}\Phi_i$ can not change during the transition.

Spatial Adjacency
- Necessary and Sufficient Conditions

- Let ${}^{AB}\Phi_i$ be more constrained than ${}^{AB}\Phi_j$ and let and ${}^{AB}\Phi_i$ satisfy the necessary condition. Qualitative configurations ${}^{AB}\Phi_i$ and ${}^{AB}\Phi_j$ are adjacent at $q^* \in P_k({}^{AB}\Phi_i)$ if and only if there exists a twist t_1 such that:

$$({}^{AB}d_{mn})^t t_1 = 0 \qquad \forall (m,n) \in \{(m,n) \mid [{}^{AB}\Phi_j]_{mn} = 0\}$$

$$({}^{AB}d_{mn})^t t_1 > 0 \qquad \forall (m,n) \in \{(m,n) \mid [{}^{AB}\Phi_i]_{mn} = 0 \wedge [{}^{AB}\Phi_j]_{mn} = +1\}$$

$$({}^{AB}d_{mn})^t t_1 < 0 \qquad \forall (m,n) \in \{(m,n) \mid [{}^{AB}\Phi_i]_{mn} = 0 \wedge [{}^{AB}\Phi_j]_{mn} = -1\}$$

Contact Description

- A contact description between two non-convex polyhedra A and B is the union $\gamma^{A,B}$ of the contact descriptions between combinations of convex polyhedra A_i and B_j of the convex decompositions of A and B:

$$\gamma^{A,B} = \bigcup_{\substack{1 \le i \le m_{cv}^{A} \\ 1 \le j \le m_{cv}^{B}}} \gamma^{A_i, B_j}$$

Contact Information Extraction

- Vertex-face contacts

  Let A_i and B_j be convex polyhedra. There is contact between vertex $v_k^{A_i}$ and face $f_l^{B_j}$ if and only if

$$ {}^{A_i B_j}\phi_{kl}^{vf} = 0 $$

$$ (\forall f_m^{B_j} \in adj(f_l^{B_j}))(({}^{A_i B_j}\phi_{km}^{vf} = -1) \vee ({}^{A_i B_j}\phi_{km}^{vf} = 0)) $$

Contact Information Extraction (cont')

- Face-vertex contacts

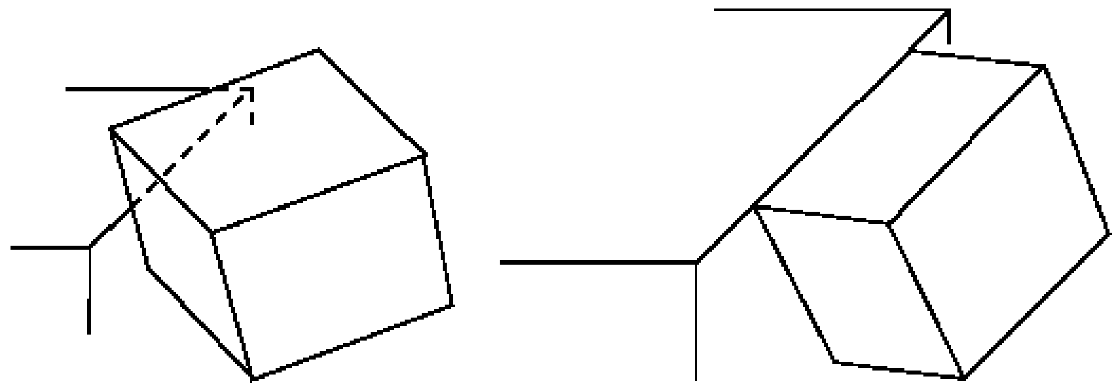
  Let A_i and B_j be convex polyhedra. There is contact between face $f_k^{A_i}$ and vertex $v_l^{B_j}$ if and only if

$$^{A_iB_j}\phi_{kl}^{fv} = 0$$

$$(\forall f_m^{A_i} \in adj(f_k^{A_i}))((^{A_iB_j}\phi_{ml}^{fv} = -1) \vee (^{A_iB_j}\phi_{ml}^{fv} = 0))$$

Contact Information Extraction (cont')

- Edge-edge contacts



(a) (b)

Two types of edge-edge contacts are considered: *edge-edge-cross* contacts and *edge-edge-touch* contacts. An edge-edge-cross contact exist when two edges contact each other but are not parallel. This is shown in Frame (a). An edge-edge-touch contact exist when two edges contact each other while they are parallel. This is shown in Frame (b).

Contact Information Extraction (cont')

- Edge-edge-cross contact

  Let A_i and B_j be convex polyhedra. There exists and edge-edge cross contact between edge $e_k^{A_i}$ and $e_l^{B_j}$ if and only if the following three conditions are satisfied:

$$^{A_iB_j}\phi_{kl}^{ee} = 0$$

$$({}^{A_iB_j}\phi_{k_1l_1}^{vf} \neq {}^{A_iB_j}\phi_{k_2l_1}^{vf}) \vee ({}^{A_iB_j}\phi_{k_1l_2}^{vf} \neq {}^{A_iB_j}\phi_{k_2l_2}^{vf})$$

$$({}^{A_iB_j}\phi_{k_1l_1}^{fv} \neq {}^{A_iB_j}\phi_{k_1l_2}^{fv}) \vee ({}^{A_iB_j}\phi_{k_2l_1}^{fv} \neq {}^{A_iB_j}\phi_{k_2l_2}^{fv})$$

Contact Information Extraction (cont')

- Edge-edge-touch contact

  Let A_i and B_j be convex polyhedra. There exists and edge-edge touch contact between edge $e_k^{A_i}$ and $e_l^{B_j}$ if and only if the following two conditions are satisfied:

$$({}^{A_iB_j}\phi_{k_1l_1}^{vf}=0\wedge{}^{A_iB_j}\phi_{k_1l_2}^{vf}=0)\wedge({}^{A_iB_j}\phi_{k_2l_1}^{vf}=0\wedge{}^{A_iB_j}\phi_{k_2l_2}^{vf}=0)$$

$$({}^{A_iB_j}\phi_{k_3l_1}^{fv}={}^{A_iB_j}\phi_{k_4l_1}^{fv}=-1)\vee({}^{A_iB_j}\phi_{k_3l_2}^{fv}={}^{A_iB_j}\phi_{k_4l_2}^{fv}=-1)\vee$$

$$({}^{A_iB_j}\phi_{k_1l_3}^{vf}={}^{A_iB_j}\phi_{k_1l_4}^{vf}=-1)\vee({}^{A_iB_j}\phi_{k_2l_3}^{vf}={}^{A_iB_j}\phi_{k_2l_4}^{vf}=-1)$$

A Simple Metric for Planning

- Define the *cost-to-go* from ${}^{AB}\phi_i$ to ${}^{AB}\phi_{goal}$ along trajectory $T_{tr}({}^{AB}\phi_i, {}^{AB}\phi_{goal})$ as:

$$CTG(T_{tr}({}^{AB}\phi_i, {}^{AB}\phi_{goal})) = n - \sum_{k=0}^{n-1} \beta N_c({}^{AB}\phi_{i+k})$$

Summary:
Discrete Event Controller

- Synthesize-Discrete-Event-Controller algorithm integrates the methods for hypothesis generation, hypotheses testing and contact information extracting into one algorithm that synthesizes a discrete even controller.
- The search should be terminated if an acceptable path from the goal configuration to the start configuration has been found or if all states have been expanded.

Motion Planner

- Assembly motion planning results from the consideration of uncertainty during the assembly process.
- Several works on this problem:
  - Local strategies vs global motion planning
  - LMT framework vs two phase approach

Local Strategies

- Related works:
  - A passive compliant device for peg-in-hole problem – by researchers at Charles Stark Draper lab.
  - A biased-search strategy and a tilt-and-slide strategy for peg-in-hole problem – by Inuoe.
  - A tilting strategy for convex pegs – by Strip.
  - A tilting strategy for inserting rectangular pegs – by Caine.

Local Strategies (cont’)

- Characteristic: purely local, help planning the path which the grasped part should move in response to the forces/torques of reaction.
- Problem: do not consider the problem on a more global scale.

Global Motion Planning

- Related works:
  - Using task geometry to place constraints on the compliant motions – by Mason.
  - A planner based on a contact-space representation of the parts – by Laugier.
  - Several works in the following LMT framework and two-phase planners.

LMT Framework

- Related works:
  - Computing motion strategies given bounded uncertainty using pre-images in configuration space (C-space) – by Lozano-Perez, Mason and Taylor.
  - Plan compliant motions in a finite state space for 3D translation without rotation – by S.J.Buckley.

LMT Framework (cont')

- It attempts to form a one-step motion plan by finding a motion vector which moves the grasped part from the starting configuration to the goal configuration.
- The LMT models of uncertainty in the initial pose and in control have been followed by most other researchers.
- Limitations:
  - Computational methods for constructing pre-image have been found only for polyhedral C-space.
  - Poor computational efficiency.

Two Phase Approach

- Idea:
  - Phase 1: a nominal trajectory in free space is computed without considering uncertainty, only the geometric and spatial constraints are taken into account.
  - Phase 2: the uncertainties are taken into account to refine the nominal trajectory and assembly motion plan is developed from it.

Two Phase Approach (cont')

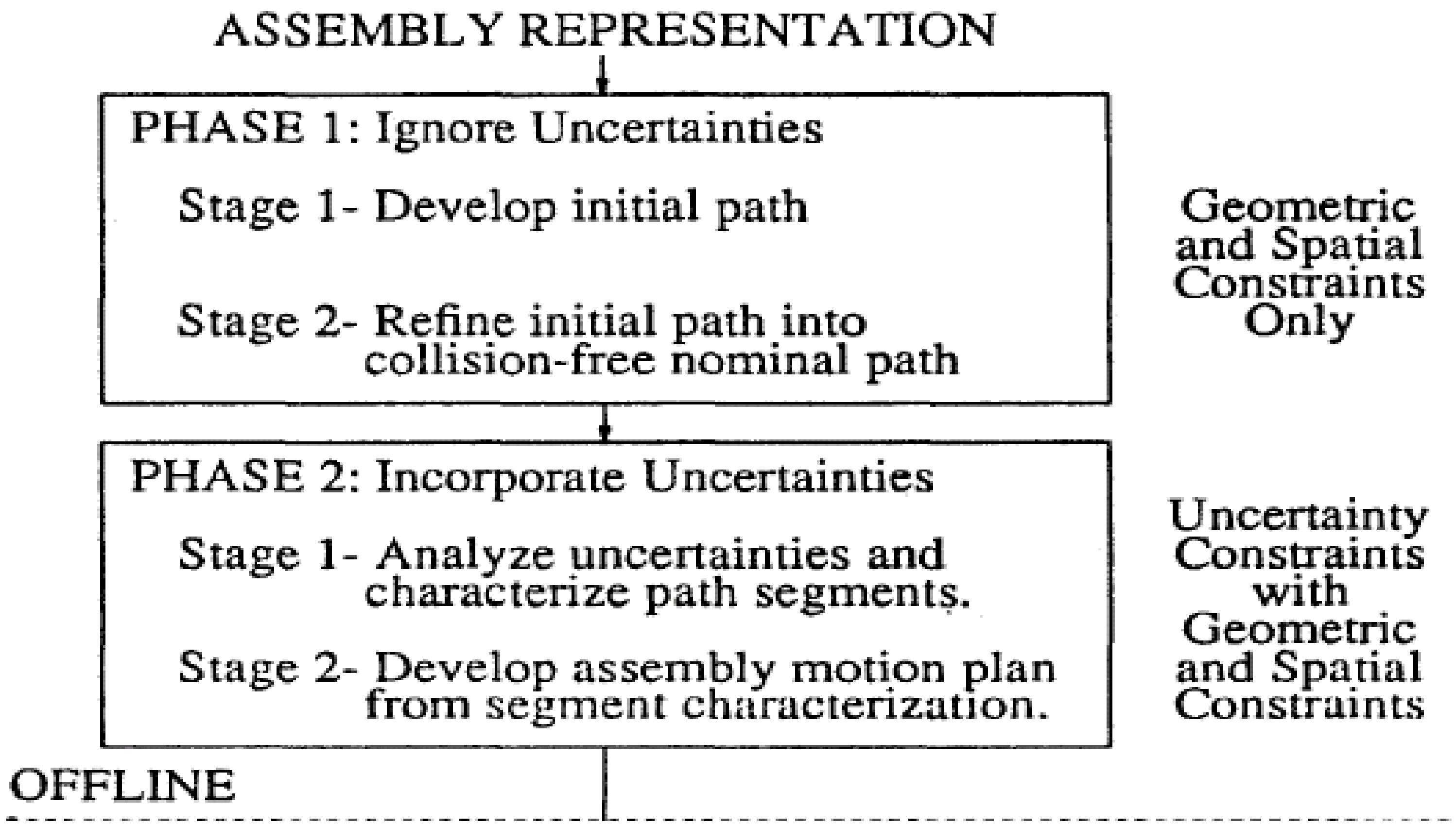
- Related works:
  - Design of nominal velocities and admittance matrices to guarantee force assembly – by Schimmels and Peshkin.
  - A fine motion planner for assembly where force/torque guided motions are incorporated into the plan as needed – by Gottschlich.
  - Find a nominal solution path in both free and contact configuration space – by Rosell, Basanez and Suarez.
  - Let a deviation from the nominal trajectory trigger an error recovery strategy or patch plan – by Xiao and Volz.

Two Phase Approach (cont’)

- Generating hypothetical contact formations – by Xiao and Zhang.
- Linearizing the configuration space around critical points – by Dakin and Popplestone.

Two Phase Approach (cont’)

- Example: - by Gottschlich



Summary:
Motion Planner

- Some related works are introduced.
- Those works are dealt with the problem of generating motion planning for assembly by different representations, sensors and approaches.
- Our work is to compute the motion planning by qualitative contact models. The motion planning is generated from the output of the discrete event controller, which will compute a qualitative contact state transition path for the assembly problem.

Robot Controller

- In order to implement robotic assembly, a robot controller will be designed to guide the grasped part to achieve the desired state transition that was specified by the discrete event controller.
- Inputs: velocity commands from motion planner.
- Outputs: commands to guide robot movements.
- Some existed compliant motion robot control laws will be considered to be implemented in the robot controller.

Conclusions and Future Work

- The whole system of robotic assembly is introduced.
- Design a motion planner to compute compliant motion parameters based on the desired contact transition generated by discrete event controller.
- Design/Find a suitable robot controller to execute the assembly tasks.
- Improve the discrete event controller by considering the hardware limitations and uncertainties.
- Synthesis above three parts to complete the whole robotic assembly system.