Spatial Logic for Retrieving Similarly Shaped Parts – Research Update

Mark Ascher - Dept of ECE

Chandan Pitta – Dept of ECE

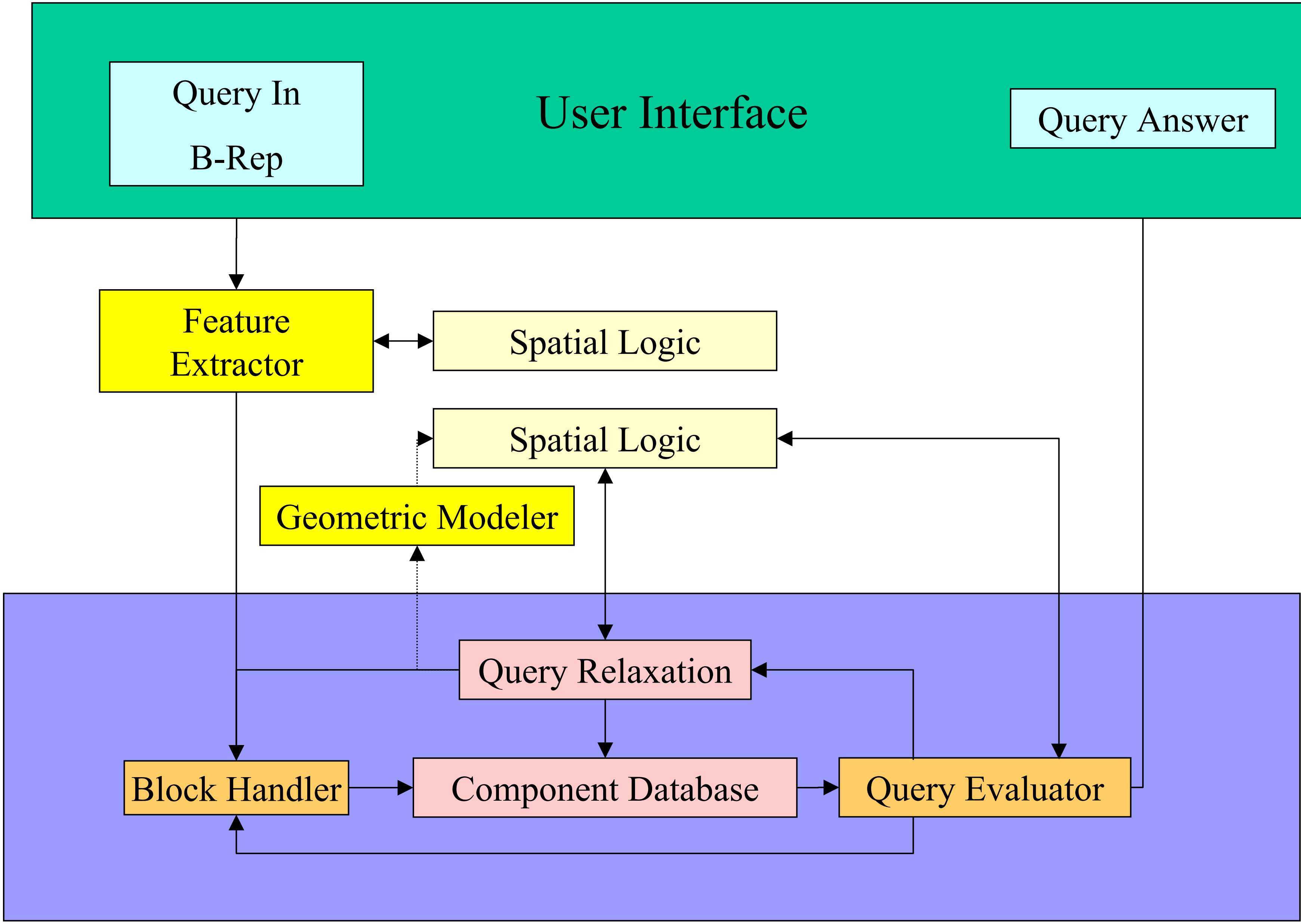
Motivation

- Retrieval of similarly shaped components can:
  - Add functionality to existing CAD databases
  - allow for the reuse of process plans which can both speed up and reduce the cost of development.

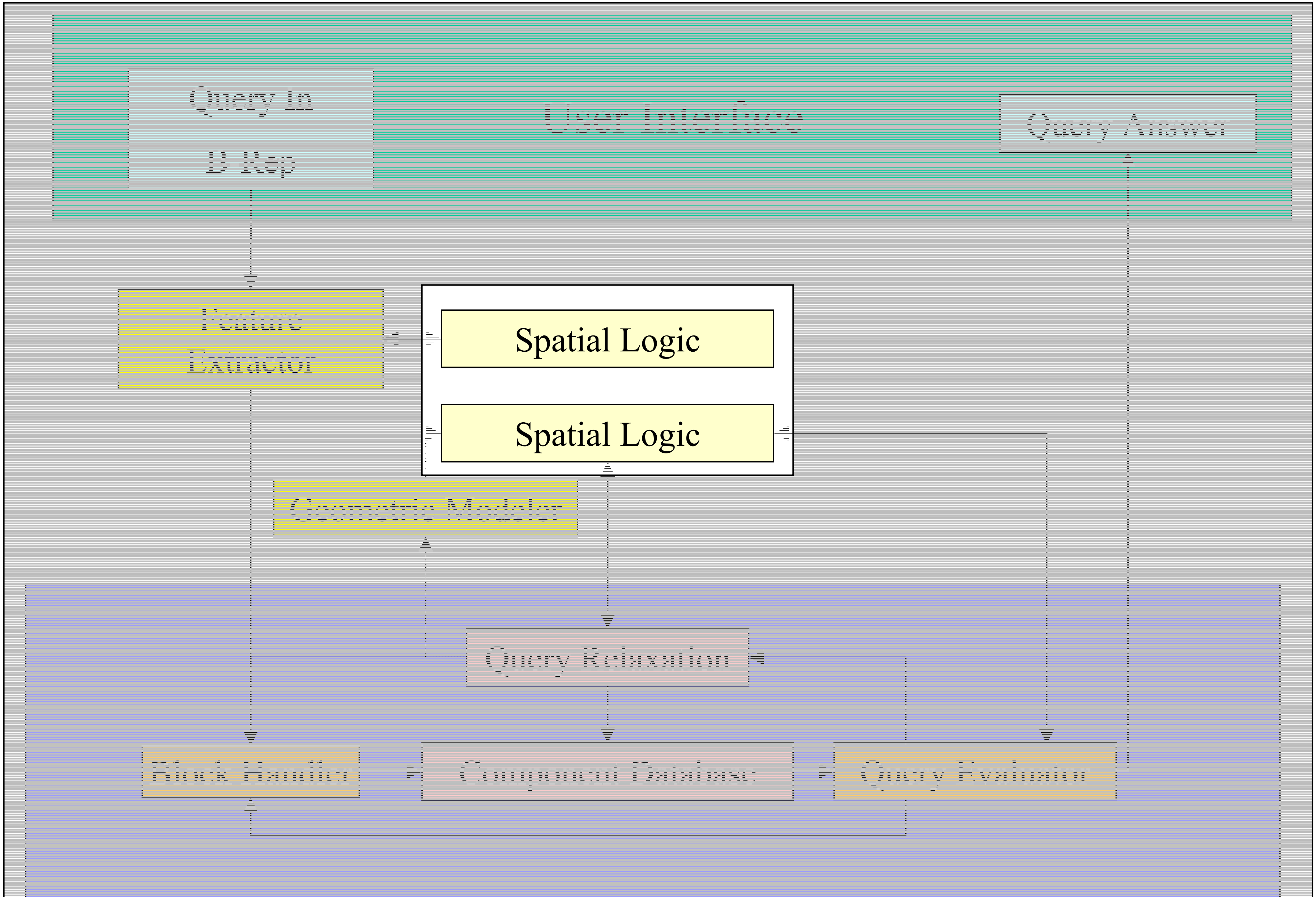
Challenges

- Retrieval of similarly shaped components has many challenges:
  - Multiple interpretations
  - Interacting features
  - Topological differences do not guarantee component dissimilarity
  - Graph matching solution is computationally intensive

System Overview



Spatial Logic



Goals of Spatial Logic

- Development of a Usable Logic for Problem of Machinable part Shape Similarity
  - Characterize feature-feature interactions
  - Reasoning about interactions
    - Determining new interactions from existing information
    - Dynamically modify interaction for relaxation
    - Use of Characterizations for clustering

Related Work

- Volumetric Reasoning
  - Lee, Scott, Williams, and Cox
- Spatial Reasoning using Region Connection Calculus
  - Randell, Cui, Cohn
  - Bennett
  - Renz Nebel

Volumetric Reasoning

- Defines volumes as a set of internal points unioned with a set of boundary points
  - $P_i(V) \cap P_b(V)$
  - Positive volumes have solid (exisiting) internal and boundary points
  - Negative Volumes have nonsolid (removed) internal points
- Absolute Volume (||V||) refers to a volume regardless of type
- An inverse is defined which compares positive and negative volumes inv(V)

Volumetric Reasoning

- Defines three Volumetric Relations
  - Inclusion of Volumes
    - V2 is included in V1 if same type or included in inv(V) if inverses
    - $V1 \otimes V2$
  - Interpenetration of Volumes
    - V1 and V2 have some volumetric overlap
    - $V1 \oplus V2 \equiv \exists V ((V1 \otimes V) \wedge (V2 \otimes V1))$
  - Adjacency of Volumes
    - V1 and V2 are adjacent if and only if the interpenetration of the volumes is a single plane
    - $V1 \mathrm{O} V2 \equiv p \in V, p \in Pb(V_1), p \in Pb(V_2)$
- Similar Definitions defined for Topological Relations

Volumetric Reasoning

- Uses defined Topological Relations to reason about volume removal in machined parts.
- Determines three types of relationships between features:
  - Inclusion
  - Interpenetration
  - Inclusion (Ownership)
- Useful when using the complete graph of a part.
  - Maximal Feature Interaction reasoning requires a richer relationship base.

Spatial Reasoning

- Defines basic relation C(x,y)
  - X connects with y
- Other Relations based on this
  - Disconnected $DC(x,y) = \neg C(x,y)$
  - Part of $P(x,y) = \forall z[C(z,x) \rightarrow C(z,y)]$
  - Proper Part of $PP(x,y) = P(x,y) \wedge \neg P(y,x)$
  - Overlaps $O(x,y) = \exists z[P(z,x) \wedge P(z,y)]$
  - Partial Overlap $PO(x,y) = O(x,y) \wedge \neg P(x,y) \wedge \neg P(y,x)$
  - Discrete from $DR(x,y) = \neg O(x,y)$
  - Tangential PP $TPP(x,y) = PP(x,y) \wedge \exists z[EC(z,x) \wedge EC(z,y)]$
  - Edge Connected $EC(x,y) = C(x,y) \wedge \neg O(x,y)$
  - Non-TPP $NTPP(x,y) = PP(x,y) \wedge \neg \exists z[EC(z,x) \wedge EC(z,y)]$

Spatial Reasoning

- Boolean Operators are also defined:
  - Sum of the two regions sum(x,y)
  - The Universal Spatial Region us
  - Complement of a region compl(x)
  - Intersection of two regions prod(x,y)
  - Difference of two regions diff(x,y)
- Other Relationships Introduced:
  - A connected one-piece region CON(x)
  - Convex region CONV(x)
  - A region is inside the convex hull of another INSIDE(x,y)
  - Part of a region is inside the convex hull of P-INSIDE(x,y)
  - A region is outside the convex hull of another OUTSIDE(x,y)

Spatial Reasoning

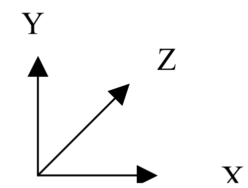
- An inferencing table is developed to estimate the interaction between two regions given their interactions with a third region
- Continuity Constraints are added to show how regions go from one relation to another
- Directional relations mentioned but not implemented
- Bennett extended the original work by tansforming the first order logic to a propositional logic
- Renz and Nebel extended the original work by adding a Modal Logic implementation

Logic for current Work

- Based on three dimensional ortho-normal features
- All features represent a volume (no lines, planes, or points)
- 3-D logic built up from 1-D information
- Qualitative reasoning supported
- All features bounded by 6 faces
- Maximal Feature Considerations
- Interactions are directed
  - Feature order matters
  - Determined in both directions for current uses

Fundamental Definitions

- Bottom = Lowest Y Value
- Top = Highest Y Value
- Left = Lowest X Value
- Right = Highest X Value
- Front = Lowest Z Value
- Back = Highest Z Value
- A feature can be considered two sets of faces {L,H}
  - L = $\forall Fa\ \exists F(((Fa = Bottom) \vee (Fa = Left) \vee (Fa = Front)))$
  - H = $\forall Fa \exists F(((Fa = Top) \vee (Fa = Right) \vee (Fa = Back)))$



Fundamental 1-D Comparitors

- Equal
  - The two faces are at the same point in the direction of interest
- Greater Than
  - The face is not on the same half-space of the face being compared as the center of the feature
- Less Than
  - The face is on the same half-space of the face being compared as the center of the feature

One Dimensional Face Relations

- Define how one face is related to the cross section of a feature in one dimension
- Definitions
  - FF1 indicates the feature face on the same side of the feature as the face being compared (if Fa in L FF1 in L)
  - FF2 indicates the feature face on the opposite side of the feature as the face being compared (if Fa in L FF2 in H)

One Dimensional Face Relations

- Using the convections and comparators described the following Face Relations can be defined:
- Outside Other (OO): $(OO) \Rightarrow (Fa > FF2)$
- Same Other (SO): $(SO) \Rightarrow (Fa = FF2)$
- Inside Both (IB): $(IB) \Rightarrow (Fa < FF1) \wedge (Fa > FF2)$
- Same Like SL: $(SL) \Rightarrow (Fa = FF1)$
- Outside Like (OL): $(OL) \Rightarrow (Fa > FF1)$

One Dimensional Interactions

- Combining the face relation knowledge for the parallel faces of a feature defines the one dimensional interaction
- Results in 25 (5x5) potenial 1D interactions
- All combinations not physically meaningful
  - 13 Physically Possible
  - 8 unique phisically meaningful combinations
  - 1 of these is disconnected

Value	OO	SO	IB	SL	OL
OO	X	X	X	X	N
SO	X	X	X	X	
IB	X	X			
SL	X	X			
OL	N				

One Dimensional Interactions

- There are 8 Physical Configurations that can exist
- Disconnected (D):
  - D $\Rightarrow \exists$ OO
- Edge Connected (EC):
  - EC $\Rightarrow \exists$ SO
- Partial Overlap (PO):
  - PO $\Rightarrow$ ((Fa1 OL) $\wedge$ (Fa2 IB)) $\vee$ ((Fa1 IB) $\wedge$ (Fa2 OL))
- Edge Connected Sub Part (ECB)
  - ECB $\Rightarrow$((Fa1 SL) $\wedge$ (Fa2 IB)) $\vee$ ((Fa1 IB) $\wedge$ (Fa2 SL))
- Equal (Eq)
  - EQ $\Rightarrow$ (Fa1 SL) $\wedge$ (Fa2 SL)
- Super Part (SP)
  - SP $\Rightarrow$((Fa1 OL) $\wedge$ (Fa2 OL)) $\vee$ ((Fa1 OL) $\wedge$ (Fa2 OL))
- Edge Connected Super Part (ECP)
  - ECP $\Rightarrow$((Fa1 SL) $\wedge$ (Fa2 OL)) $\vee$ ((Fa1 OL) $\wedge$ (Fa2 SL))
- Sub Part (BP)
  - BP $\Rightarrow$ (Fa1 IB) $\wedge$ (Fa2 IB)

Qualitative Interaction Families

- Determined by the one-dimensional interactions
- Used as a clustering method for determining similarity

Value	OO	SO	IB	SL	OL
OO	X	X	X	X	N
SO	X	X	X	X	Face
IB	X	X	Non-determining	Non-determining	Improper
SL	X	X	Non-determining	Non-determining	Intrusion
OL	N	Face	Improper	Intrusion	Pass-Through

Three Dimensional Interactions

- The three dimensional interactions between features are defined by the combination of the three one-dimensional interactions.
- Using maximal features limits the combinations that can exist.
- Allows the three-dimensional interaction to be qualitatively identified.

Three Dimensional Interactions

- Combining the three One-D Interactions (7 interacting combinations) results in (7x7x7) 343 3-dimensional interactions.
- From a qualitative perspective many of these are not unique. Results in 84 unique combinations of 3 1D Relationships

Three Dimensional Interactions

- Use of Maximal Features further reduces acceptable combinations
  - There cannot be more than 2 from the set {SP, ECB, EQ} in any interaction
  - There cannot be more than one EC in an interaction
  - If 2 from {SP, ECB, EQ} then the third must be in the set {E, SP}
  - There must be at least one from {SP, ECB, PO}
  - There must be at least one from {PO, ECP, SP}
  - There cannot be more than one EQ in an interaction
  - If any dimension is disconnected the features are disconnected.
  - If 2 from {EQ, ECP, SP} then third must be from {SP, ECB}

Three Dimensional Interactions

- There cannot be more than 2 from the set {SP, ECB, EQ} in any interaction
  - All of these are inside the feature area therefor the third must be outside
- There cannot be more than one EC in an interaction
  - More than one results in an edge or vertex interaction
- If 2 from {SP, ECB, EQ} then the third must be in the set {E, SP}
  - Since 2 are inside the third must be outside
- There must be at least one from {SP, ECB, PO}
  - At least one face must be outside or the feature is subsumed

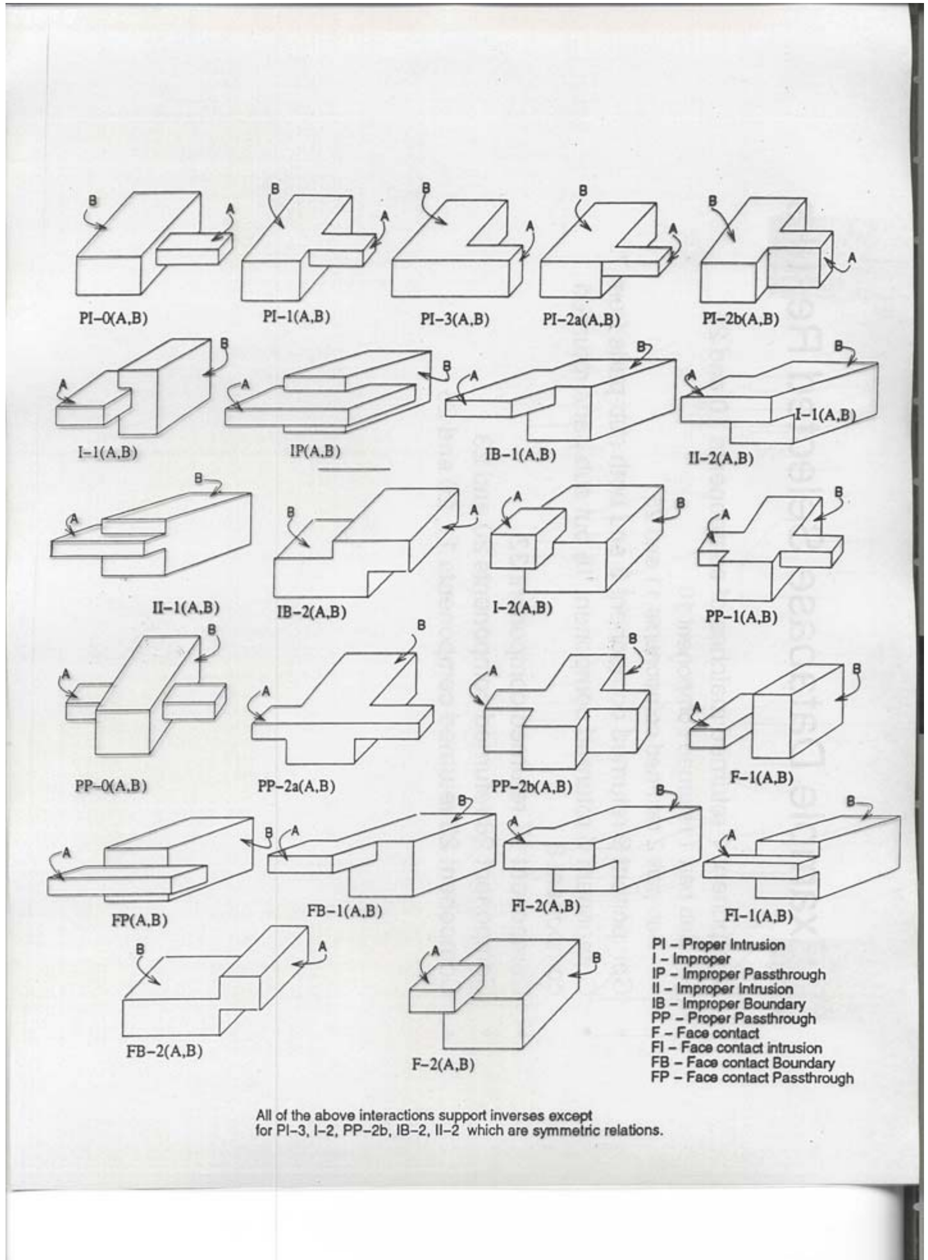
Three Dimensional Interactions

- There must be at least one from {PO, ECP, SP}
  - There must be at least one face inside or the feature is subsuming (complement of previous rule)
- There cannot be more than one EQ in an interaction
  - Two EQ results in the same 2D Cross section therefore the two features create one maximal-volume
- If any dimension is disconnected the features are disconnected.
  - If any one dimension does not interact than the others are not in the same space as the feature of interest
- If 2 from {EQ, ECP, SP} then third must be from {SP, ECB
  - Since 2 are outside third must be inside

Three Dimensional Interactions

- The Maximal Feature Constraints reduce the total from 84 down to 36 unique 3D directional interactions
- These are the Cannonical interactions and there inverses
- Total reduction of potential 3-D Interactions = 307
  - 259 for redundant qualitative interactions
  - 48 for Maximal Feature Constraints
- Include 23 Interactions on next slide with their inverses

Show All Interactions Here



Reasoning with Spatial Logic

- Determining New Iteractions from Existing ones
  - Requires use of Face-Pair information for better resolution since qualitative 1D interactions lose direction information
  - Uses Transitive Properties of Face relations

Value	03	11	12	13	21	22	23	30	31	32	33
03	2y	N	30	3y	N	30	3y	N	N	30	3y
11	N	xx	x3	x3	3x	33	33	N	3x	33	33
12	03	yz	y2	y3	3z	32	33	N	3z	32	33
13	1y	yz	y1	yy	3z	31	3y	N	3z	31	3y
21	N	zy	z3	z3	2y	23	23	30	3y	33	33
22	03	11	12	13	21	22	23	30	31	32	33
23	1y	11	11	1y	21	21	2y	30	31	31	3y
30	N	N	N	N	03	03	03	y2	y3	y3	y3
31	N	zy	z3	z3	1y	13	13	y1	yy	y3	y3
32	03	11	12	13	11	12	13	y1	y1	y2	y3
33	1y	11	11	1y	11	11	1y	y1	y1	y1	yy

x – U y – >O z – <S N – no interaction (OO)

0 =SO, 1 = IB, 2 = SL, 3 = OL

Example 1 of Reasoning

- Given Features a,b and c and I(a,c) and I(b,c) find I(b,a)
- I(a,c)= BP = IB,IB, I(b,c) = ECP = SL,OL
- Since maxc < mina < minc and
- minb = minc Implies mina < minb
- Since minc < maxa < maxc and
- maxb > maxc Implies maxa < maxb
- Therefore I(b,a) = OL, OL = SP

Example 2 of Reasoning

- Given Features a,b and c and I(a,c) and I(b,c) find I(b,a)
- I(a,c) = ECB = IB,SL, I(b,c) = PO = IB,OL
- Since maxc < mina < minc and
- maxc < minb < minc Can not Tell relation between mina and minb
- Since maxa = maxc and maxb < maxc Implies maxa > maxb
- Know that minb < maxa so minb < maxa < maxb so Interaction Exists
- Therefore I(b,a) = {OL, SL, IB}, OL = {SP, ECP, PO}

Example 3 of Reasoning

- Given Features a,b and c and I(a,c) and I(b,c) find I(b,a)
- I(a,c) = BP = IB,IB, I(b,c) = PO = IB,OL
- Since maxc < mina < minc and
- maxc < minb < minc Cannot Tell mina Relation to minb
- Since minc < maxa < maxc and
- maxb > maxc Implies minb < maxa < maxb
- Since minc < maxa < maxc and minc < minb < maxc can not tell if the two features interact However We do know that the possibilities are:
  - mina < maxa < minb implies no interaction
  - mina < maxa = minb implies Face type interaction
  - mina < minb < maxa implies Improper interaction
  - mina = minb < maxa Implies Intrusion Interaction
  - minb < mina < maxa Implies Passthrough

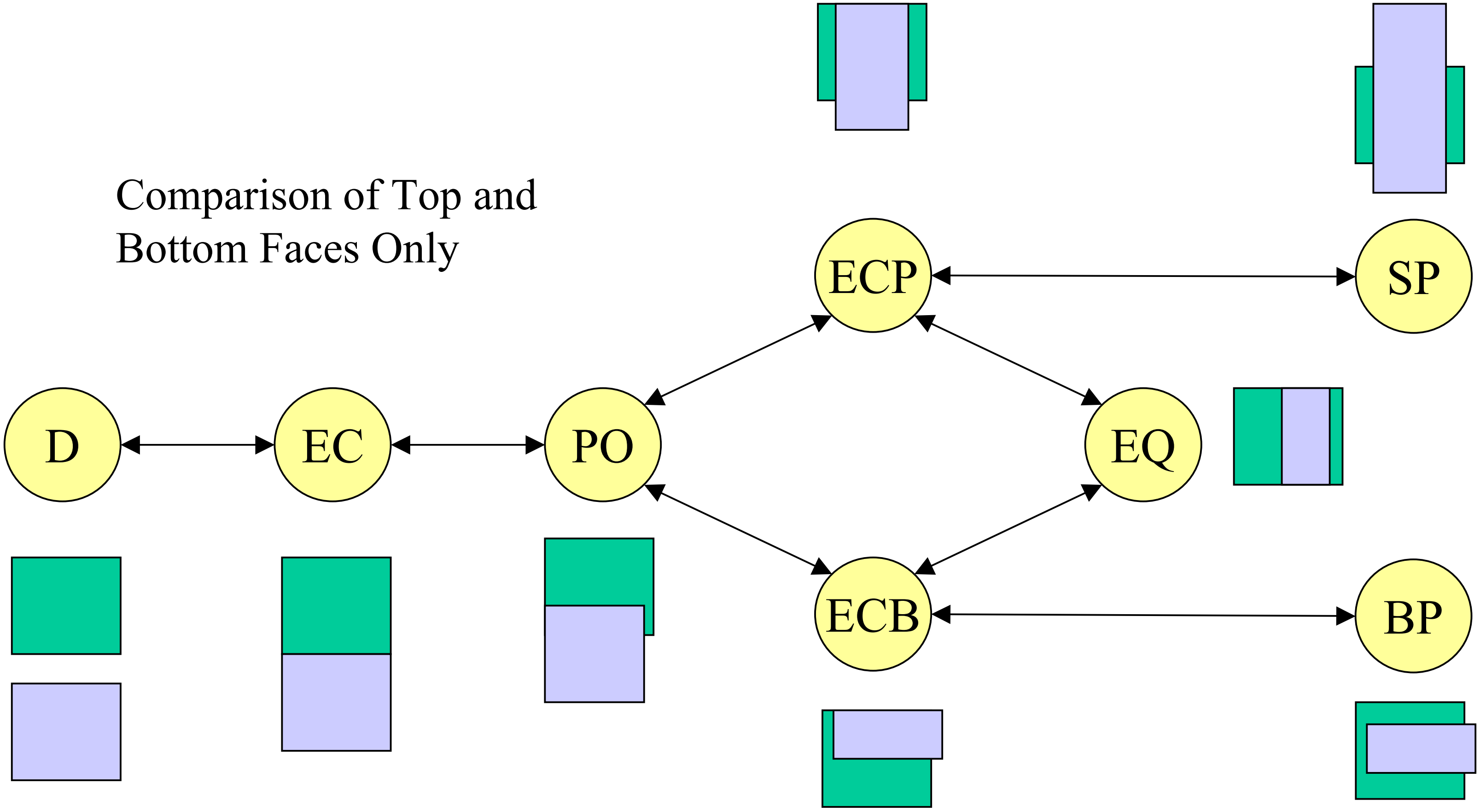
Relaxation With Spatial Logic

- Relaxation of query part is to be done a face by face basis.
  - Need to know if face is “relaxable”
    - Boundary faces can not be relaxed beyond edge of component
    - Can not violate Maximal Feature Constraints
  - Relies on Qualitative Ordering of Interactions (Next Slide)
    - Only one face relaxed at a time
    - A face can only make one qualitative step at a time

1D Qualitative Ordering of Interactions

- The following Graph shows Ordering of interactions

Comparison of Top and Bottom Faces Only

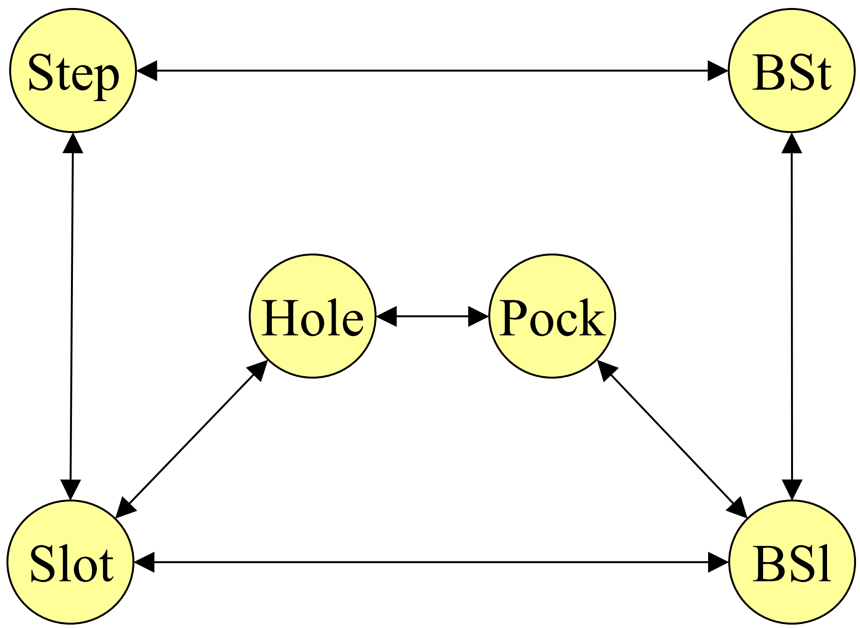


Relaxation of Faces

- The boundary faces of a feature define the feature type
  - Relaxation changes the feature type as well as the interaction
  - Relaxation of a boundary face can only be into a component
- If a boundary face is made, need to make sure the feature is still valid

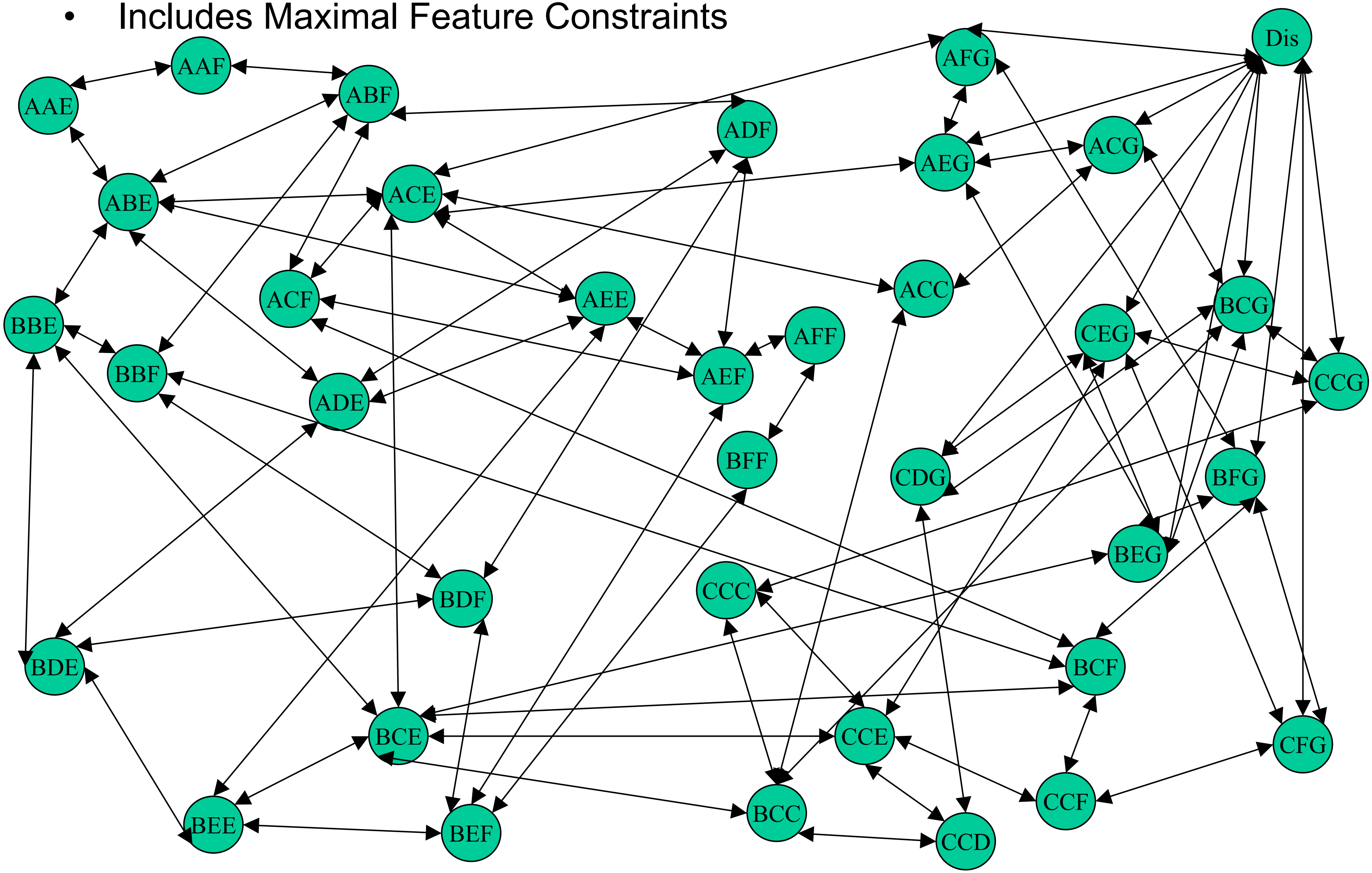
Qualitative Ordering of Features

- The following Graph shows how relaxation of feature boundary faces can change a feature type



3D Interaction Relaxation

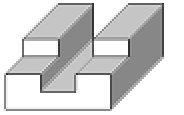
- Combine 1D interactions to generate 3D Graph
- Includes Maximal Feature Constraints



Face Relaxation Algorithm

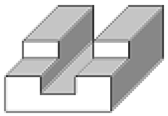
- Pick Face
  - Determine Candidate Relaxations
    - If Boundary then
      - Determine relaxable direction
      - Determine resulting feature
    - Use Interaction Ordering with any boundary constraints
  - Check Maximal Feature Constraints
  - Return list of relaxed interactions and feature type

Example



- F1 = Step, F2 = Slot, Interaction = PP2a
- PP2a Corresponds to BBF on previous Graph
- If Pick Bottom of Step
  - Not a boundary face
  - Can Move up or down
    - If move Up Next qualitative step removes feature
    - Qualitative move down becomes even with bottom of slot
      - BBF Changes to BDF = PP3
      - Return step, PP3
- If Pick Front of Step
  - Boundary face
    - Can only move Back
    - Qualitative Move Back Changes Feature to a Slot
      - BBF Changes to ABF = PP1
      - Return Slot, PP1

Example



- F1 = Step, F2 = Slot, Interaction = PP2a
- PP2a Corresponds to BBF on previous Graph
- If Pick Right Side of Step
  - Boundary face
    - Can only move Left
    - Qualitative Move Left Changes Feature to a Blind Step
      - BBF Changes to BBE = PI2a
      - Return Blind Step, PI-2A

Conclusions

- Spatial Logic Developed for use in both determining and characterizing interactions
- Spatial Logic can be used to inference about the interactions between features based on their mutual interactions with a third feature

Future Work

- Query Relaxation Unit
  - Two methods being considered
- Query Evaluation Unit
  - Perform graph matching on the candidate matches
- Implement Spatial Logic with Query Relaxation

System Demonstration

- Chandan will demonstrate the system in the lab for anyone interested