Motion Planning for Component Manipulation/Assembly

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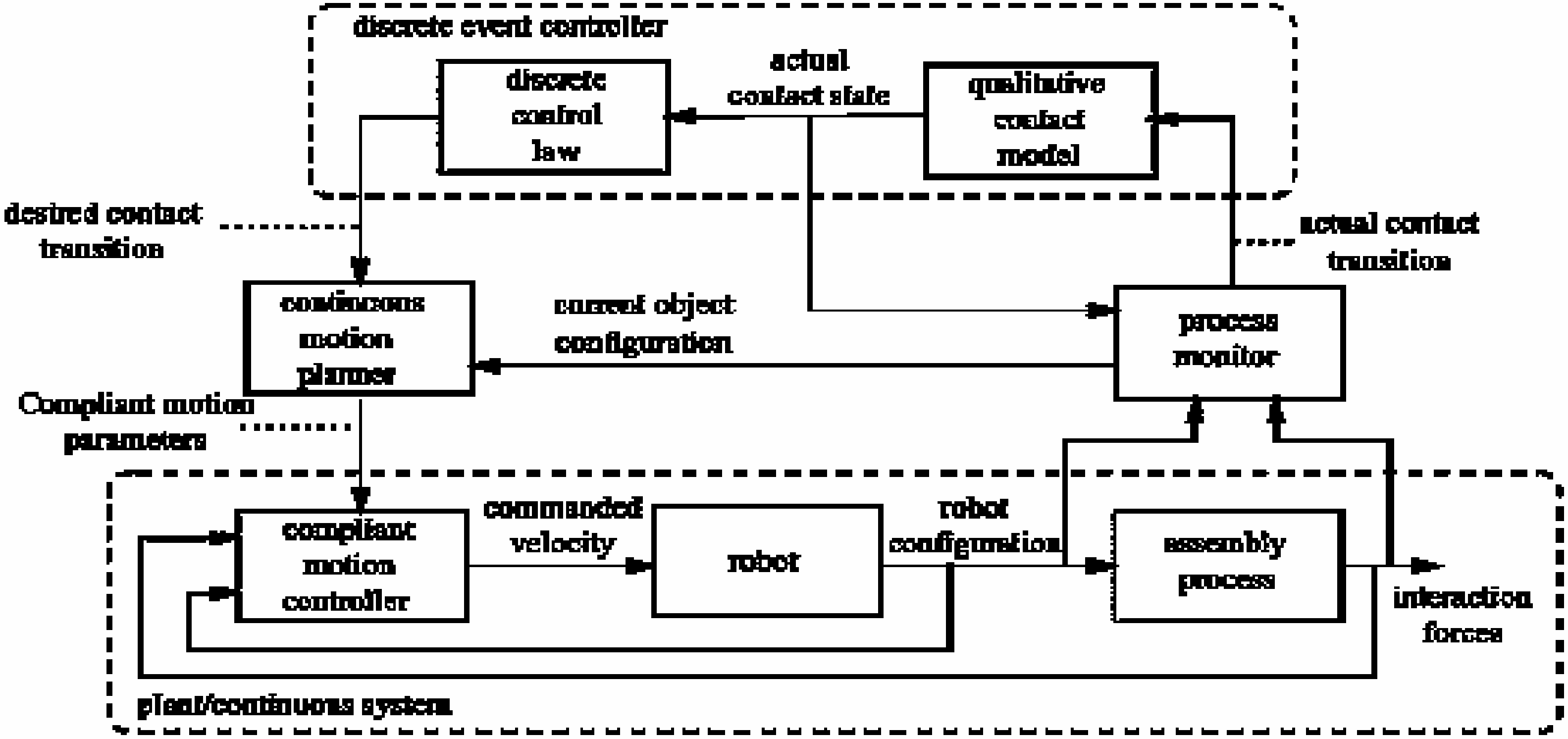
03/12/04

Automatic Assembly System



System Components

- Process Monitor
- Discrete Event Controller
- Compliant Motion Planner
- Robot Controller

Compliant Motion Planning – Problem Statement

- Given desired contact transition, i.e., the current contact state and the next desired contact state, and current object configuration, how to generate the compliant motion parameters for the robot controller to guide the work piece to the next desired contact state.

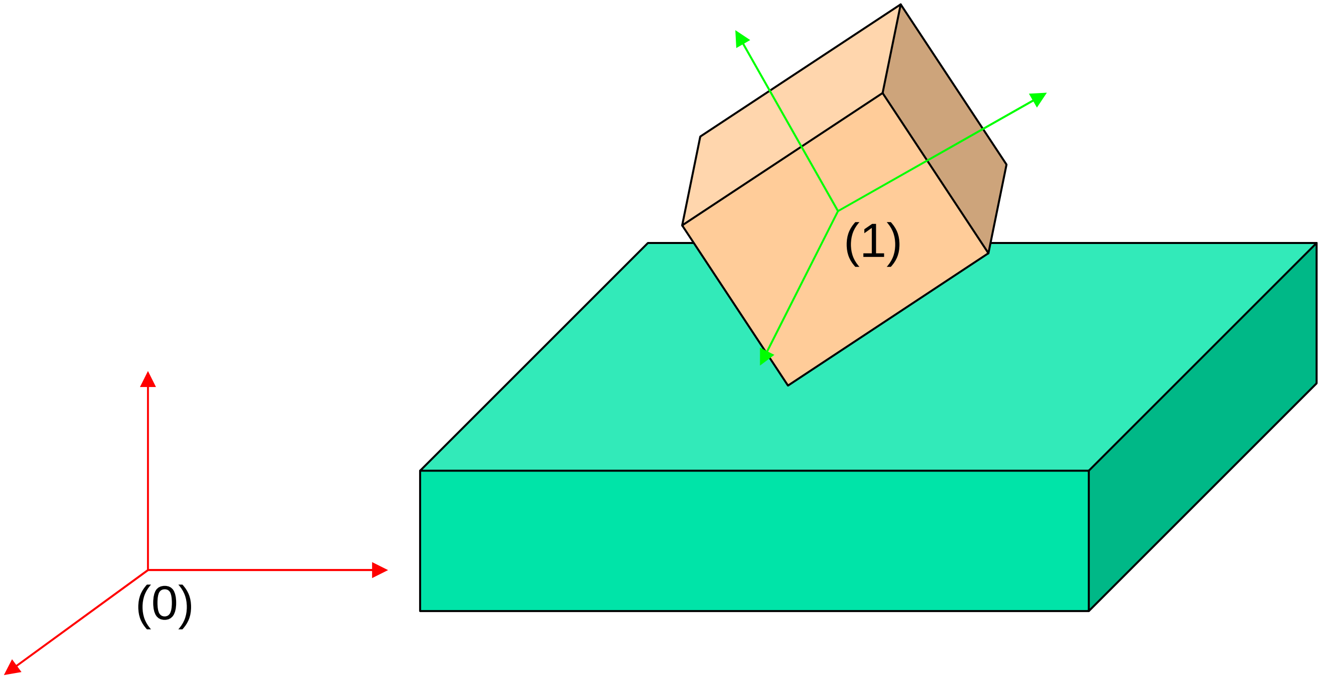
Compliant Motion Planner - Contents

- Basic concepts.
- Three types of contact and their geometric constraints.
- Feature interaction matrix.
- Motion control conditions.
- Optimization problem and upper bound.
- Future work.

Contact between Two Polyhedra

- Assumption: One object B (fixture) is assumed to be fixed while the other object A (workpiece) is mobile.
- There are three types of elementary contacts:
  - Vertex-face contacts : $v_i^A - f_j^B$
  - Face-vertex contacts: $f_i^A - v_j^B$
  - Edge-edge contacts: $e_i^A - e_j^B$
- A contact description between two polyhedra A and B is described by a set of elementary contacts.
- A *qualitative contact state*, or *contact state* for short, is a geometrically feasible description.

Coordinate Frames



- It is convenient to define two coordinate frames:
- The moving coordinate frame (1) is conveniently fixed to the moving object and is called the body-fixed reference frame. The origin O of the body-fixed frame is usually chosen to coincide with the center of gravity (CG) when CG is in the principal plane of symmetry or at any other convenient point if this is not satisfied.
- Another coordinate frame is an inertial reference frame (0).

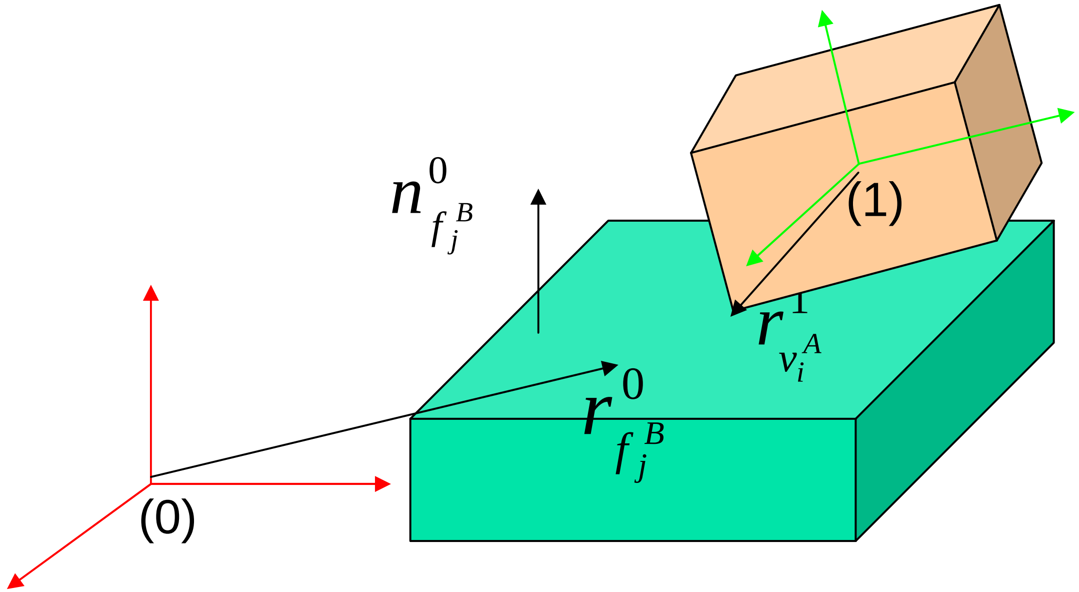
Configuration of Moving Object

- A configuration of moving object A is defined as (r_1^0, R_1^0) .
- where r_1^0 is the position of the body-fixed frame with respect to the inertial frame, and R_1^0 is the transformation matrix from body-fixed frame (1) to inertial frame (0), which describe the orientation of body-fixed frame with respect to the inertial frame.
- Then the generalized coordinate of the workpiece is defined as a (6×1) vector:

$$q = [x \quad y \quad z \quad \alpha \quad \beta \quad \theta]^T$$

- Where $r_1^0 = [x \quad y \quad z]^T$ and R_1^0 is function of $\varphi = [\alpha \quad \beta \quad \theta]^T$

Vertex-face Contact and Its Kinematic constraint

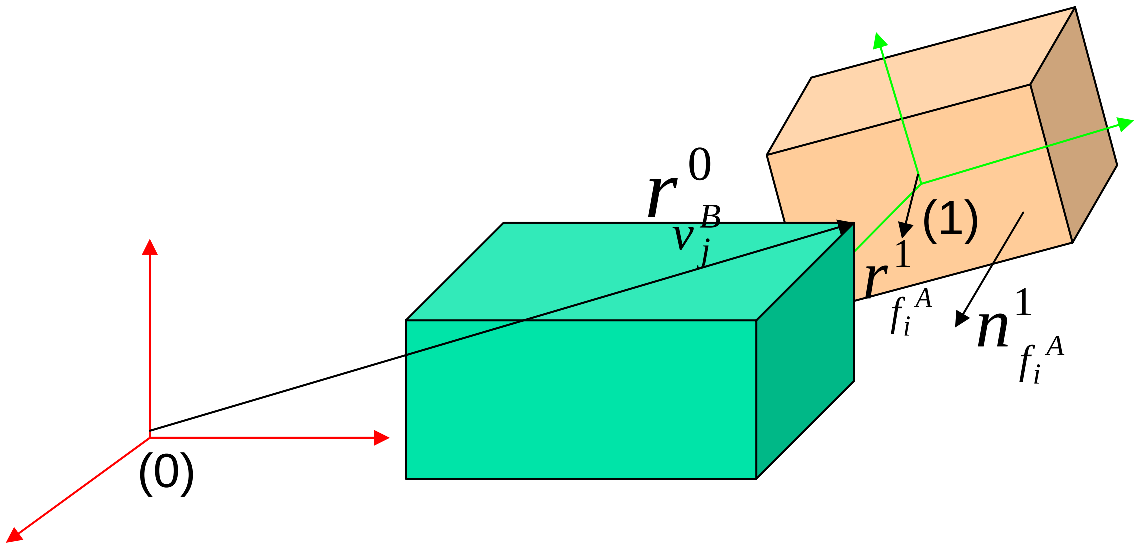


Kinematic constraint: $d_s^E(v_i^A, f_j^B) = 0$

The Euclidean distance is computed as:

$$d_s^E(v_i^A, f_j^B) = (r_1^0 + R_1^0 r_{v_i^A}^1 - r_{f_j^B}^0)^t n_{f_j^B}^0$$

Face-vertex Contact and Its Kinematic constraint

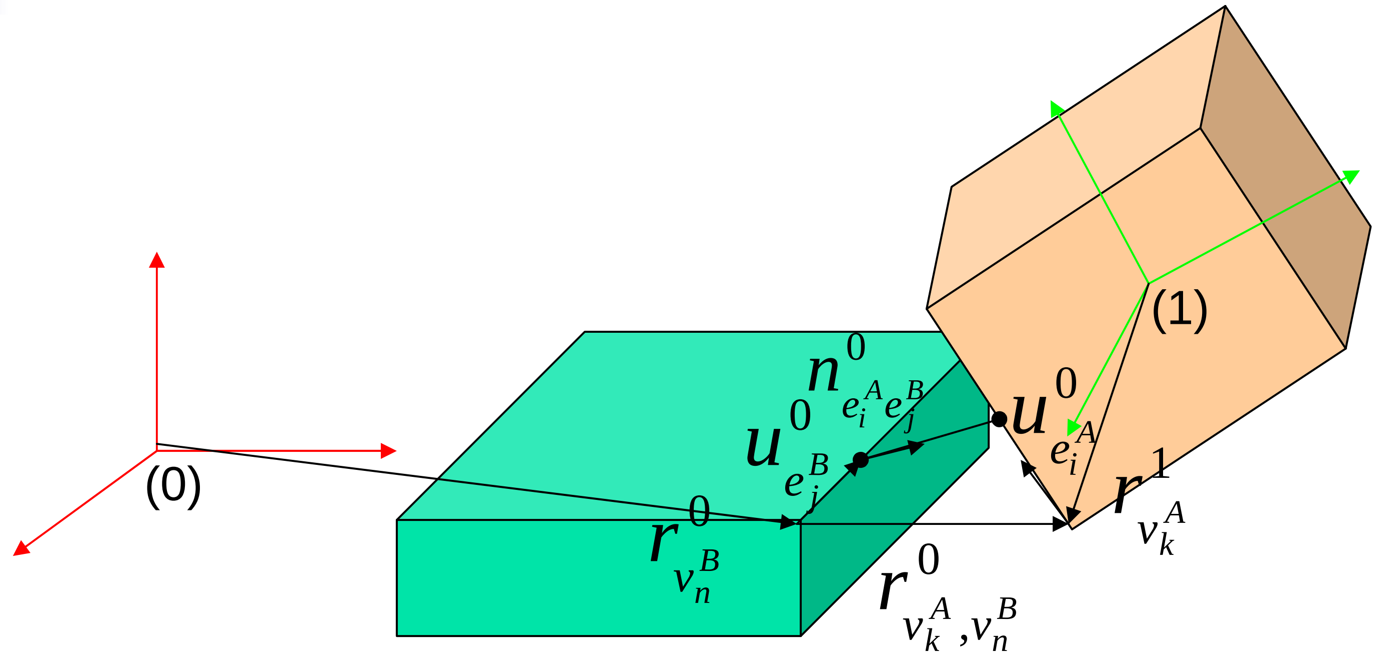


Kinematic constraint: $d_s^E(f_i^A, v_j^B) = 0$

The Euclidean distance is computed as:

$$d_s^E(f_i^A, v_j^B) = [(R_1^0)^t (r_{v_j^B}^0 - r_1^0) - r_{f_i^A}^1]^t n_{f_i^A}^1$$

Edge-edge Contact and Its Kinematic constraint



Kinematic constraint: $d_s^E(e_i^A, e_j^B) = 0$

The Euclidean distance is computed as: $d_s^E(e_i^A, e_j^B) = (n^0_{e_i^A e_j^B})^t (r^0_{v_k^A, v_n^B})$

$$= \left(\frac{1}{\left\| \tilde{u}^0_{e_i^A} u^0_{e_j^B} \right\|} \tilde{u}^0_{e_i^A} u^0_{e_j^B}\right)^t (r_1^0 + R_1^0 r^1_{v_k^A} - r^0_{v_n^B})$$

Bounding Constraints

- Vertex-face contact:

$$h_{bd}^{I}(f_j^B, v_i^A) \overset{def}{=} \prod_{f_c^B \in cv(f_j^B)} \Big[\sum_{p_k^B \in bp(f_c^B)} \mu(-d_s^E(p_k^B, v_i^A)) \Big] = 0$$

- Face-vertex contact:

$$h_{bd}^{II}(v_i^B, f_j^A) \overset{def}{=} \prod_{f_c^A \in cv(f_j^A)} \Big[\sum_{p_k^A \in bp(f_c^A)} \mu(-d_s^E(v_i^B, p_k^A)) \Big] = 0$$

- Edge-edge contact:

$$h_{bd}^{III}(e_j^B, e_i^A) \overset{def}{=} \mu(\alpha_{e_j^B e_i^A}^{B}) + \mu(l_{e_j^B} - \alpha_{e_j^B e_i^A}^{B}) + (\alpha_{e_j^B e_i^A}^{A}) + \mu(l_{e_i^A} - \alpha_{e_j^B e_i^A}^{A}) = 0$$

Qualitative Configuration representation

- Feature interaction matrix is used to describe the interaction of features of two objects A and B, which is the relative configuration of the object A with respect to object B qualitatively.
- The feature interaction matrix of non-convex objects A and B is the composition of interaction matrices between the convex decomposition of object A and convex decomposition of object B.
- The feature interaction matrix of convex objects is defined by:

$$
{}^{AB}\Phi = \begin{bmatrix} {}^{AB}\Phi^{vf} & 0 & 0 \\ 0 & {}^{AB}\Phi^{fv} & 0 \\ 0 & 0 & {}^{AB}\Phi^{ee} \end{bmatrix}
$$

- Where ${}^{AB}\Phi^{vf}$, ${}^{AB}\Phi^{fv}$ and ${}^{AB}\Phi^{ee}$ are vertex-face, face-vertex and edge-edge interaction matrices respectively.

Feature Interactions

- There are three types of feature interactions: vertex-face, face-vertex and edge-edge interactions.
- Element of vertex-face interaction is defined by: ${}^{AB}\phi_{ij}^{vf} = sign(d_s^E(v_i^A, f_j^B))$
- Element of face-vertex interaction is defined by: ${}^{AB}\phi_{ij}^{fv} = sign(d_s^E(f_i^A, v_j^B))$
- Element of edge-edge interaction is defined by: ${}^{AB}\phi_{ij}^{ee} = sign(d_s^E(e_i^A, e_j^B))$

Motion Control Conditions

- There are four conditions for the motion planning based on the kinematic constraints of three types elementary contacts.
- Maintaining condition: ensures that the currently active constraints will be held, if desired.
- Enabling condition: ensures that the next desired transition is allowed to occur.
- Disabling condition: ensures that an undesired transition is not allowed to occur.
- Geometric condition: ensures that the geometric constraints are satisfied.

Maintaining Condition

- For the zero element in current state (feature interaction matrix), if it is still zero in the next desired state (feature interaction matrix), it means we should keep this type of contact.
- To keep the desired contact, the absolute Euclidian distance d_s^E of this type of contact must keep zero, i.e., $\frac{d}{dt} d_s^E = 0$

Enabling Condition

- For the non-zero element in current state (feature interaction matrix), if it is changed to zero in the next desired state (feature interaction matrix), it means we gain of this type of contact.
- To gain the desired contact, the absolute Euclidian distance d_s^E of this type of contact must decrease, i.e.,

$$\left| \frac{d(d_s^E)}{dt} \right| < 0$$

- Define a scalar γ as: $\gamma = {}^{AB}\phi_{ij}$, where ${}^{AB}\phi_{ij}$ is for current state.
- The enabling condition is: $\gamma \frac{d(d_s^E)}{dt} < 0$

Disabling Condition

- For the zero element in current state (feature interaction matrix), if it is changed to non-zero in the next desired state (feature interaction matrix), it means we loss of this type of contact.
- To loss the undesired contact, the absolute Euclidian distance d_s^E of this type of contact must increase, i.e., $\left|\frac{d(d_s^E)}{dt}\right| > 0$
- Define a scalar γ as: $\gamma = -{}^{AB}\phi_{ij}$, where ${}^{AB}\phi_{ij}$ is for next desired state.
- The enabling condition is: $\gamma \frac{d(d_s^E)}{dt} < 0$

Geometric Condition

- In order to reach the desired transition, all kinds of contact changes must be realized at the same time.
- We are not interested in the magnitude of the velocity command but its direction.
- Suppose that the time required is Δt , which means after Δt , the new configuration of moving object must satisfy all the geometric constraints of the next desired contact state.

Velocity Parameters

- The generalized coordinate of the work piece is defined as a (6×1) vector:

$$q = [x \quad y \quad z \quad \alpha \quad \beta \quad \theta]^T = \begin{bmatrix} r \\ \varphi \end{bmatrix}$$

- A configuration of moving object A is defined as (r_1^0, R_1^0)
- Velocity parameters is defined by v_1^0 and ω_1^0, where $v_1^0 = \dot{r}_1^0$ and $\omega_1^0 = \dot{\varphi}_1^0$ and

$$\dot{q} = \begin{bmatrix} \dot{r}_1^0 \\ \dot{\varphi}_1^0 \end{bmatrix} = \begin{bmatrix} v_1^0 \\ \omega_1^0 \end{bmatrix}$$

. Also we have $\dot{R}_1^0 = \omega_1^0 R_1^0$

Vertex-face Contact
- Differential of Euclidian Distance

- From the equation of distance

$$d_s^E(v_i^A, f_j^B) = (r_1^0 + R_1^0 r_{v_i^A}^1 - r_{f_j^B}^0)^t n_{f_j^B}^0$$

we can get

$$\delta d_s^E(v_i^A, f_j^B) = (\delta r + \delta\tilde{\varphi} R_1^0 r_{v_i^A}^1)^t n_{f_j^B}^0 = (n_{f_j^B}^0)^t \delta r + (\tilde{r}_{v_i^A}^{0,1} n_{f_j^B}^0)^t \delta\varphi$$

- So we have

$$\frac{\delta d_s^E(v_i^A, f_j^B)}{\delta q} = \begin{bmatrix} n_{f_j^B}^0 \\ \tilde{r}_{v_i^A}^{0,1} n_{f_j^B}^0 \end{bmatrix}$$

Face-vertex Contact
- Differential of Euclidian Distance

- From the equation of distance

$$d_s^E(f_i^A, v_j^B) = [(R_1^0)^t(r_{v_j^B}^0 - r_1^0) - r_{f_i^A}^1]^t n_{f_i^A}^1$$

we can get

$$\delta d_s^E(f_i^A, v_j^B) = (n_{v_i^A}^1)^t[(\delta\tilde{\varphi}R_1^0)^t(r_{v_j^B}^0 - r_1^0) - (R_1^0)^t\delta r]$$

$$= -(n_{f_i^A}^0)^t\delta r - (\tilde{r}_{v_j^B}^0 - \tilde{r}_1^0)n_{f_i^A}^0)^t\delta\varphi$$

- So we have $\dfrac{\delta d_s^E(f_i^A, v_j^B)}{\delta q} = \begin{bmatrix} -n_{f_i^A}^0 \\ -(\tilde{r}_{v_j^B}^0 - \tilde{r}_1^0)n_{f_i^A}^0 \end{bmatrix}$

Edge-edge Contact
- Differential of Euclidian Distance

- From the equation of distance

$$d_s^E(e_i^A, e_j^B) = \left(\frac{1}{\left\| \tilde{u}^0_{e_i^A} u^0_{e_j^B} \right\|} \tilde{u}^0_{e_i^A} u^0_{e_j^B}\right)^t (r_1^0 + R_1^0 r^1_{v_k^A} - r^0_{v_n^B})$$

we can get

$$\delta d_s^E(e_i^A, e_j^B) = [\frac{\partial}{\partial r} d_s^E(e_i^A, e_j^B)]^t \delta r + [\frac{\partial}{\partial \varphi} d_s^E(e_i^A, e_j^B)]^t \delta \varphi$$

- So we have

$$\frac{\delta d_s^E(e_i^A, e_j^B)}{\delta q} = \begin{bmatrix} \frac{\partial}{\partial r} d_s^E(e_i^A, e_j^B) \\ \frac{\partial}{\partial \varphi} d_s^E(e_i^A, e_j^B) \end{bmatrix}$$

where $\frac{\partial}{\partial r} d_s^E(e_i^A, e_j^B) = n^0_{e_i^A e_j^B}$

$$\frac{\partial}{\partial \varphi} d_s^E(e_i^A, e_j^B) = (\tilde{r}^0_{v_k^A} - \tilde{r}_1^0) n^0_{e_i^A e_j^B} - \frac{1}{\left\| \tilde{u}^0_{e_j^B} u^0_{e_i^A} \right\|} \tilde{u}^0_{e_i^A} \tilde{u}^0_{e_j^B} [I_3 - n^0_{e_i^A e_j^B} (n^0_{e_i^A e_j^B})^t] r^0_{v_k^A, v_n^B}$$

Moving Object Configuration

- After Δt time, the new configuration of the moving object will be:

  $$(r_1^0 + \dot{r} \cdot \Delta t, R_1^0 + \dot{\omega} \cdot R_1^0 \cdot \Delta t)$$

- It should satisfy both the kinematic and bounding constraints of the desired contact state.

Motion Planning

- Differential of Euclidian distance:

$$\frac{d}{dt}d_s^E = \frac{d}{dq}d_s^E\,\frac{dq}{dt} = \frac{\delta d_s^E}{\delta q}\dot{q}$$

- The motion planning problem is equal to find velocity parameters $\dot{q}$ such that maintaining, enabling, disabling and geometric conditions are satisfied.
- There will be more than one result satisfying the conditions, so we can define an objective function to obtain an optimal result.

Optimization Problem

- Define $l = \sqrt{(x_1^0)^2 + (y_1^0)^2 + (z_1^0)^2}$
- Optimization problem:

$$Min \qquad J = \dot{x}^2 + \dot{y}^2 + \dot{z}^2 + (l\dot{\alpha})^2 + (l\dot{\beta})^2 + (l\dot{\theta})^2$$

$S.t$:

maitaining, enabling, disabling and geometric constraints are satisfied.

Here, let $\Delta t = 1$.

Upper Bound

- When find the optimal solution, setting an upper bound by

$$[\dot{x}^2 + \dot{y}^2 + \dot{z}^2 + (l\dot{\alpha})^2 + (l\dot{\beta})^2 + (l\dot{\theta})^2]\cdot\kappa^2 = 1$$

where $\kappa > 0$

- So the real output velocity parameters are:

$$\dot{q} = \begin{bmatrix} \dot{x}/\kappa & \dot{y}/\kappa & \dot{z}/\kappa & \dot{\alpha}/\kappa & \dot{\beta}/\kappa & \dot{\theta}/\kappa \end{bmatrix}^T$$

Future Work

- Find a way to solve the optimization problem.
- Programming and testing the proposed method.